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Response Functions*

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Abstract

Imagine that John must choose between two uncertain payoff distributions, knowing that the set of possible payoffs is the same for both, but nothing about the shapes of the distributions. In the first period he chooses either alternative and experiences a payoff as a result of his choice. Given this experienced payoff, in the second period he decides whether to choose the same alternative again, or switch. We model John's second period choice with a *response function*, i.e., a mapping from obtained payoffs to the probability of choosing the same alternative in the second period. We first provide results on (i) how the shape of the response function affects both expected payoffs and exposure to risk, and (ii) what standard models of choice under uncertainty would predict about the shape of the response function. We then run an experiment to elicit subjects' response functions, empirically characterize the heterogeneity across subjects with a mixture model, and illustrate how payoffs vary across response function types. Finally, we use our theoretical results, along with additional information that we collected from subjects, to interpret their response functions.

Key words: Adaptive learning, response functions, experimental economics, stochastic dominance.

JEL codes: D81, D83.

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1 Introduction

Imagine that John walks up to two slot machines. By dropping a coin in either machine he will receive a prize of between one and five gold bars. However, he knows nothing about the probability distribution over prizes for either machine. John has exactly two coins in his pocket. Upon choosing a machine for the first coin, and experiencing the corresponding prize, he must now decide whether to drop his second coin in the same machine, or switch. While simple, this environment approximates important economic decisions in which virtually no information is known ex-ante about the shape of payoff distributions, so direct experience (while limited) may have a large effect on future choice.[1,2]

Since the first period choice is made with no information, our analysis treats it as exogenously given. On the other hand, the second period choice may depend on the payoff experienced as a result of the first period choice. Thus, we represent the second period choice as a Bernoulli distribution in which the probability of staying with the same alternative is determined by the first period payoff. This representation defines a *response function*, i.e., a function mapping the experienced payoff to the probability of choosing the same alternative in the second period.

In principle, the response function is a belief-free model of choice, similar to those found in adaptive-learning models of choice, such as Easley and Rustichini (1999), Börgers et al. (2004), Mengel and Rivas (2012), Oyarzun and Sarin (2013), and Agastya and Slinko (2015). It can be viewed as the "primitive" reaction of an individual to obtained payoffs, in a problem in which decisions are made not "from description" of how likely the payoff consequences of different alternatives are, but "from experience" (cf. Erev and Haruvy (2013)). An alternative interpretation that we will also consider is that the response function is a consequence of a more structured (belief-based) decision approach such as Bayesian Expected Utility Theory.[3]

Regardless of whether one interprets the response function as a belief-free model of choice, or as having an underlying Bayesian structure, the following fundamental questions arise in the setting we consider: when people are so poorly informed, and have so little opportunity to learn, does it even matter how they respond? In particular, are there general patterns in the way that one particular response function or another affects payoffs? It turns out that there are. We first analyze: (i) how the shape of the response function affects both expected payoffs and exposure to risk, and (ii) what standard models of choice under uncertainty would predict

[1] A few examples are the choice of realtor when selling a house, of a lawyer for litigation, or of a school for each child. Similarly, investment decisions, such as whether to participate in the IPO of a certain company and subsequent secondary offerings, may not be available often, giving investors only limited opportunities to learn from their own experiences.

[2] Experimental evidence indicates that decision-makers may often have a clearer understanding of what outcomes may occur than the probability distributions over those outcomes (see, e.g., Loomes (1998), Selten et al. (1999)), and systematically over-weight experienced relative to observed information (see, e.g., Maniadis and Miller (2012), Simonsohn et al. (2008)).

[3] Under this interpretation, in the second period the individual chooses the alternative that yields the highest expected utility according to her updated beliefs.

about the shape of the response function. We then run an experiment to elicit subjects' response functions, empirically characterize the heterogeneity across subjects with a mixture model, and illustrate how payoffs vary across response function types. Finally, we use our theoretical results, along with additional information that we collected from subjects, to interpret their response functions.[4]

We begin our theoretical analysis in Section 2.1 by assuming that payoff distributions can be ordered according to stochastic dominance. Proposition 1 establishes the intuitive result that if an individual's response function is increasing (concave), then the alternative she chooses in the second period is more likely to first-order (second-order) stochastically dominate the unchosen alternative.[5] Then, we drop the assumption that distributions are ordered according to stochastic dominance, and provide an analogous result. In particular, Proposition 3 shows that if an EU-maximizer's response function is an affine transformation of her Bernoulli utility function, then the alternative she chooses in the second period is more likely to be the alternative that she would prefer if both payoff distributions were known.

Given the benefits that can arise from the increasingness of an individual's response function, one can ask whether stretching response functions to increase their slope can result in further improvements in economic performance. Accordingly, Proposition 2 shows that if an individual's response function is of *maximum strength*, meaning that it stays (switches) with probability one if the obtained payoff is the highest (lowest), then no other response function can outperform it in the second period for all pairs of payoff distributions. Further, Corollary 1 shows that no response function outperforms all others for all pairs of payoff distributions. Finally, Corollary 2 shows that an EU-maximizer whose response function is an affine transformation of her Bernoulli utility function is expected to draw from her preferred payoff distribution most often when the affine transformation is stretched to maximum strength.

Finally, as an alternative to the belief-free interpretation, we consider the possibility that an individual's

[4]A natural question is to what extent choice, in the setting that we consider, can be rationalized in terms of axioms on the response function. While, in principle, this exercise would be helpful for developing descriptively-better models of preferences over ambiguous alternatives, the extreme minimality of information in our setting rules out the possibility of verifying the axioms that arise in relatively richer information environments (e.g., Anscombe and Aumann (1963), Gilboa and Schmeidler (1989), Klibanoff et al. (2005)). For instance, Anscombe and Aumann (1963) assume that when individuals choose from ambiguous lotteries that differ only in the prize associated with one state of the world, then preferences over lotteries are fully determined by preferences over prizes in that state. By contrast, because in our setting individuals lack the type of information available in theirs (i.e., how acts map states to prizes), we are unable to provide an analogous axiom. Similarly, the belief-free approaches referenced earlier tend to also rely on a relatively greater availability of information. For example, there it is typical to consider an individual who acquires information over time, by repeated experience, which is not possible in the one-shot learning environment that we consider. To illustrate, Easley and Rustichini (1999) consider axioms regarding how preferences over alternatives change in response to observed states, and assume "exchangeability," i.e., that upon observing two states, preferences over alternatives should be the same, regardless of what order the states are observed in. An analogous axiom is not possible in our setting because the individual observes only one payoff realization. On the other hand, the analyses of Börgers et al. (2004) and Oyarzun and Sarin (2013), as explained below, relate more closely to our approach.

[5]It has long been observed in the psychology and computer sciences literatures that learning models which increase the probability of choosing those alternatives that have been successful, have the property that the "ex ante" expected value of the probability of choosing the alternative that is most likely to result in a success is increasing in time (see, e.g., Norman (1968) and Narendra and Thathachar (1974)). These insights were first introduced to economics by Börgers et al. (2004), and Oyarzun and Sarin (2013) extended their analysis to include risk aversion.

response function is supported structurally by Bayesian reasoning. Observation 2 of Section 2.2 characterizes Bayesian John's response function in terms of his preferences and prior beliefs. His response function is *extreme* in the sense of staying with the same alternative with probability one for some payoffs, and with probability zero for the others,[6] though it need not be monotonic. Corollary 3 (of Appendix B) provides plausible conditions on prior beliefs under which his response function is also (weakly) monotonically increasing. Lastly, in Section 2.3 we introduce a stochastic component to Bayesian John's choice process using the *random parameter* approach (c.f., Apesteguia and Ballester (2017)), and show that under standard assumptions response functions are smooth, with slopes that we characterize and interpret.[7]

We then run an experiment to study individuals' *behavior* in the minimal information setting that we consider, and provide a first glance at the empirical distribution of response functions across subjects. While our theoretical analysis of payoff performance, as well as the response functions implied by Bayesian decision-making, raise a number of hypotheses of interest, those arising from the former analysis are less shaped by ancillary assumptions on unobservable characteristics of individuals, such as beliefs. In particular the results on performance that we summarize above highlight conditions under which it is advantageous to have an increasing response function, and in particular, a maximum strength increasing response function.[8] These results invite the question of whether the response functions elicited from individuals' choice behavior have these properties. On the other hand, regarding our Bayesian results, the spectrum of possible beliefs is wide, making the set of possible predictions from the structural analysis large and varied. Nevertheless, in Section 6 we discuss how certain plausible classes of beliefs can provide a rationale for some of the behavior that we observe.

In order to elicit individuals' response functions, we observe subjects face a choice problem that is very similar to the slot machine example given above. In the first round of each task subjects choose one of two alternatives, and then experience its corresponding payoff. In the second round, after experiencing the first round payoff, subjects must again choose one of the same two alternatives, but this time they do not observe the payoff. A subject's payoffs are determined by her choices in both the first round and the second round. Before starting, subjects are informed that, for each alternative in each such *task*, its distribution of payoffs is identical in the first and second rounds, and that the obtained payoff may be either 1, 2, 3, 4, or 5 Euros. In addition, they are told that each alternative's payoff distribution changes from one task to the next (in the example given above imagine both slot machines being replaced between tasks). Subjects are not told anything

[6]The only exception is that for some rare payoffs John can be indifferent between staying or switching, or any probability-mixture of the two.

[7]In Appendix C we provide a similar analysis under the *random utility approach*, characterizing the slope of the response function in Proposition 4. In Appendix D we show how to extend the analysis to allow for ambiguity aversion.

[8]Similarly, concave response functions result in less risky payoffs than their convex, or linear, counterparts. In our empirical analysis we also consider a weaker notion of concavity that, under certain conditions, leads to less risky choices.

else about the distributions. Each subject makes choices in 40 such tasks.[9] We estimate the response functions of subjects using the relative frequency of tasks in which subjects choose the same alternative in both rounds, for each experienced payoff.[10]

In order to empirically characterize the heterogeneity in behavior across subjects, we use a mixture model, which yields several *classes* of response function.[11] We find that behavior in more than half of the subjects is best fit by an increasing response function, and for the majority of these subjects, one of near maximum strength. Among these subjects, seemingly minor differences in the shape of the response function have non-negligible effects on the riskiness of round 2 payoffs. The remainder of subjects are less responsive (to varying degrees) to the payoffs experienced in round 1.

Although subjects do not know the payoff distributions of each alternative, the pairs of distributions used in each task have a clear stochastic dominance relationship. In half of the decision tasks, one alternative first-order stochastically dominates the other, and in the other half, one alternative second-order stochastically dominates the other.[12] This feature of the design allows us to illustrate (i) the significantly and substantially higher average second round payoffs of subjects with increasing (and, in particular, near maximum-strength) response functions, relative to those of other subjects, and (ii) how a form of local concavity in a certain class of response function can lead to a significant and substantial decrease in the riskiness of round two payoffs, relative to those of an otherwise similar class of response function.

Finally, we briefly explore possible determinants of subjects' response functions. First we consider how observed differences across individuals might allow one to predict an individual's response function class. We find that the demographic variables sex, GPA, major, year in college, and education level of parents have no effect on this assignation. On the other hand, what appears to be evidence of the deeply rooted behavioral bias "the Gambler's Fallacy" (see, e.g., Tversky and Kahneman (1971), Rabin (2002), and Miller and Sanjurjo (2015)), observed in subjects' answers to an incentivized pre-experiment comprehension test, has a significant effect. We use this result, along with existing evidence on anticipated regret, and simple predictions of our

[9]The reason why we do not show subjects their second round payoff is to avoid giving them feedback about the performance of their response function. In contrast to most experimental studies in learning, our purpose is not to observe how individuals learn through repetitions of a certain decision task, but rather to estimate subjects' response functions by observing repetitions of their second round choices in response to their first round payoffs alone.

[10]The repeated experience of payoffs across tasks could, in principle, lead individuals to form beliefs about possible payoff distributions. In our experiment, however, this seems unlikely, given that (i) subjects are told that the distributions change after each task (in an unknown manner), and (ii) the range of possible distributions for two alternatives, even with only five possible outcomes, is vast. Further, if such belief formation were to have taken place, then one might expect to see a shift, or some type of convergence, in choice behavior across tasks; however, when we test for this, splitting the sample into the first and last 20 tasks, we see no evidence of a change in behavior across the split, either at the individual or pooled level (for details, see Appendix G).

[11]The number of classes is determined endogenously, by the Bayesian Information Criterion.

[12]In addition, in Section 3 we explain how our selection of pairs of payoff distributions across tasks was done in such a way that the elicitation of each subject's response function was not sensitive to the specific relationships existing therein, and that it was not possible for subjects to infer what these relationships were. Thus, the response functions we elicit are of more general interest in the sense that they can be considered as primitives, whereby one can use them to perform the counterfactual exercise of exploring the payoff consequences of subjects' choice behavior in an environment characterized by any pair of (unknown) payoff distributions.

theoretical analysis in order to provide some candidate interpretations of observed response function classes.

Our study sheds light on real world learning problems in environments in which (i) only minimal information is available, but one directly experiences outcomes, or (ii) more information is available, but one relatively over-weighs direct experience. This systematic over-weighting of direct experience can lead to considerable economic consequences. For example, Choi et al. (2009) find that individual investors who experience high returns on their savings increase their saving rates compared to other investors, Kaustia and Knüpfer (2008) find similar results for Finnish investors' subscriptions to IPO's, and Benartzi (2001) finds that investors over-invest more in company stock following periods of success than periods of failure.[13] By conducting a controlled laboratory experiment we are able to analyze peoples' behavior across a range of variable direct experience, to relate the performance of these decisions to the properties of outcome distributions, and to empirically characterize the distribution of learning types.[14]

Related literature. As suggested by the use of slot machines in the motivating example, the setting is an extremely short-lived multi-armed bandit in which the individual is not endowed with prior beliefs. Thus, her initial choice may be regarded as "exploration" and her second (and final) choice as pure "exploitation." By contrast, the experimental literature on multi-armed bandits (see, e.g., Banks et al. (1997), Anderson (2012), Hu et al. (2013)) typically assumes that individuals are endowed with priors about the payoff distributions (and at times, some of these distributions are even known), a large number of opportunities to draw from distributions, and tends to study either the optimal moment to stop exploring and start exploiting, whether belief updating follows Bayes rule, or the effect of ambiguity aversion on individuals' choices.

The response function is more reminiscent of the class of statistical models used in psychology to describe reinforcement learning (see, e.g., Bush and Mosteller (1951, 1955)), and broadly used to study learning in games (see, e.g., Erev and Roth (1998), Feltovich (2000), Hopkins (2002)), though models of learning typically study choice repeated many times, allow individuals to experience long histories of payoff realizations, and track states of learning over such histories of experienced payoffs and choices (see, e.g., Barron and Erev (2003), Grosskopf et al. (2006), Nevo and Erev (2012), and Erev and Haruvy (2013) for a review). Whereas a setting with many repeated decisions may approximate "small decision problems" well (cf. Erev and Haruvy (2013)), in which each choice has a relatively modest consequence (e.g., grocery store purchases), the type of only once-repeated decision tasks that we consider may better approximate those real life decisions that are infrequent, but sometimes carry

[13] Benartzi (2001) argues that the effect is driven by over-inference. Another possibility may be the over-weighting of direct experience.

[14] The more complete description of the economic consequences of personal experience on individuals' decisions, made possible by our experimental approach, allows us to test hypotheses in a way that is difficult to do with field data: if individuals in the field respond aggressively to the magnitude of obtained payoffs, then the demand of assets yielding higher returns will be positively affected by experience; similarly, if individuals are likely to repeat those choices that yield "average" returns, then the demand of assets that yield less risky returns will be positively affected by experience.

large economic consequences (see footnote 1).[15]

The literature that relates most closely to our work compares the performance properties (in terms of expected payoffs) of adaptive learning models (Börgers et al. (2004)), and relate these learning models to risk aversion (Oyarzun and Sarin (2012, 2013)).[16] Our Propositions 1 and 3 are similar to Propositions 1 and 2 in Oyarzun and Sarin (2013), though the former can readily be applied to our minimal information setting in order to analyze our experimental data (see footnote 19 in Section 2.1). Our remaining results, and experimental study, have no close counterparts in the related literature.

Section 2 contains the theoretical framework and results, Section 3 the experimental design, Section 4 the estimation of classes of response functions, Section 5 the analysis of payoff performance, and Section 6 an interpretation of the observed response function classes.

2 Framework

In the first period, the individual chooses one of two alternatives. As a result of the choice, the individual receives a payoff x that takes a value within a compact interval of real numbers, $X := [\underline{x}, \overline{x}]$. Then, after experiencing the payoff, the individual faces exactly the same problem again in the second period. In particular, the individual knows that the unknown payoff distribution associated with each alternative remains fixed across periods, and that the payoff realizations across periods are independent of one another.

In this setting, the individual's choice of whether to stay with the same alternative in the second period, or switch, may depend on the payoff that she experienced in the first period. A general formal description of this dependence can be obtained by using the following mapping, called the *response function*, which describes the probability of the individual choosing the same alternative in the second period as in the first, as a function of the payoff experienced in the first period:

$$\theta : X \to [0, 1].$$

In Subsection 2.1 we study the payoff implications of the shape of the response function. The results apply to both the belief-free, or the Bayesian, interpretation of the response function. In Subsection 2.2 we provide the predictions that the Bayesian approach makes about the shape of the response function. In Subsection 2.3 we study the effect on the predicted shape of the response function of adding a stochastic component to the Bayesian approach.

[15]In our experiment we generate repeated choices from each individual in many (effectively) *identical* two-period tasks with payoff distributions that are different across tasks, as opposed to many choices in a single task with fixed payoff distributions. This feature allows us to use the relative frequencies of observed choices to estimate the individual's response function.

[16]The relationship between learning and risk aversion has also received considerable attention in psychology, see e.g., March (1996) and Denrell (2007).

2.1 Payoff performance of response functions

We analyze how the shape of the response function affects payoffs. In particular, we provide results on the potential benefits of increasingness, concavity, and maximum strength of the response function on payoffs when payoff distributions can be ordered according to stochastic dominance. Then we drop the stochastic dominance ordering assumption, and instead order payoff distributions in terms of the preferences of an Expected Utility maximizer. In this case we similarly find that response functions that are positive affine transformations of the individual's increasing Bernoulli utility function result in the preferred payoff distribution being chosen more often than not. Finally, we find that the preferred payoff distribution is chosen most often when the resulting response function is stretched to maximum strength.

We relate the shape of the response function to the payoff performance of the individual in the second period. In particular, we assume that there are two payoff distributions, F_a and $F_{a'}$. With probability $\frac{1}{2}$ the chosen and unchosen alternatives have distributions F_a and $F_{a'}$, respectively; otherwise they have distributions $F_{a'}$ and F_a, respectively.[17] Let $P_a(\theta)$ and $P_{a'}(\theta)$ be the probabilities that the individual faces the payoff distributions F_a and $F_{a'}$, respectively, in the second period, if her response function is θ. Then,

$$\begin{aligned} \mathbb{E}P_a(\theta) &= \frac{1}{2}\int_X \theta(x)dF_a(x) + \frac{1}{2}\int_X (1-\theta(x))\, dF_{a'}(x) \\ &= \frac{1}{2} + \frac{1}{2}\left(\int_X \theta(x)dF_a(x) - \int_X \theta(x)dF_{a'}(x)\right). \end{aligned} \tag{1}$$

This expression makes it straightforward to establish the following result about the shape of response functions and performance in the second period:

Proposition 1 *The following statements are equivalent: (i) θ is increasing (concave);*[18] *and (ii) the individual is more likely to face payoff distribution F_a than payoff distribution $F_{a'}$ in the second period and, more precisely,*

$$\mathbb{E}P_a(\theta) - \mathbb{E}P_{a'}(\theta) = \int_X \theta(x)dF_a(x) - \int_X \theta(x)dF_{a'}(x) \geq 0 \tag{2}$$

for every pair of distributions F_a and $F_{a'}$ such that F_a first (second)-order stochastically dominates $F_{a'}$, with strict inequality for at least one pair of such distributions.[19]

[17] All "primitive" random variables of the model are assumed to be independent: once it has been determined whether F_a or $F_{a'}$ has been assigned to the individual, the first round payoff realization that follows is assumed to be independent of the assignment of the payoff distribution. Likewise, the realization of the individual's stochastic choice of alternative following the first round payoff is also assumed to be independent of the assignment of distribution, as well as the realized first round payoff.

[18] To be precise, here we mean increasing in a non-trivial sense (i.e., there exist x and x' in X such that $x' > x$ and $\theta(x') > \theta(x)$). Analogously, we mean concavity in a non-trivial sense (i.e., there exist $\lambda \in (0,1)$, x, x' and x'' in X, with $x = \lambda x' + (1-\lambda)x''$, such that $\theta(x) > \lambda\theta(x') + (1-\lambda)\theta(x'')$).

[19] In the traditional adaptive-learning setup (see, e.g., Börgers et al. (2004) and Oyarzun and Sarin (2012, 2013)) it is assumed that the individual has a vector of probabilities of choosing each alternative that is revised upon observation of payoffs, according

The proof is an immediate consequence of (1) and basic properties of first- and second-order stochastic dominance (see, e.g., Rothschild and Stiglitz (1970)), so is omitted.

Increasingness. Proposition 1 shows that individuals whose response functions are increasing are more likely to face the first-order stochastically dominant payoff distribution (when ranking according to this criterion is possible) than individuals who choose at random (uniformly).[20]

In the same spirit, it seems intuitive that within the class of increasing response functions, if one response function is "more increasing" than another, this may allow it to yield superior payoffs. In order to operationalize this idea, we define a response function to be *maximum strength* if $\theta(\underline{x}) = 0$ and $\theta(\overline{x}) = 1$. Increasing response functions that satisfy maximum strength might be thought of as the "most increasing" response functions. The proof of our next result reveals that any increasing response function that is not maximum strength is dominated by one that "stretches" it: if an increasing response function θ is not maximum strength, then, there exist constants b and $c > 1$ such that the response function $\theta' = b + c\theta$ satisfies $\mathbb{E}P_a(\theta') \geq \mathbb{E}P_a(\theta)$ for all distributions F_a and $F_{a'}$ such that F_a first-order stochastically dominates $F_{a'}$. Our formal result, however, is stronger; it states that maximum strength is necessary and sufficient for an increasing response function not to be dominated by any other response function.

Proposition 2 *Suppose that θ is increasing. Then the following statements are equivalent: (i) θ is maximum strength, and (ii) there is no response function $\theta' \neq \theta$ such that $\mathbb{E}P_a(\theta') \geq \mathbb{E}P_a(\theta)$ for all pair of distributions F_a and $F_{a'}$ in which F_a first-order stochastically dominates $F_{a'}$.*

All proofs that are not omitted are provided in Appendix A. The proof of Proposition 2 reveals that for any two maximum strength response functions, we can find two different specifications of the pair of distributions F_a and $F_{a'}$, such that F_a first-order stochastically dominates $F_{a'}$, and so that θ performs (strictly) better than θ' for one specification, but worse for the other. Thus, the following result is obtained, which states that there is no "best" response function in terms of performance.

to revision rules that may vary across different alternatives. In that setup one could obtain equation (1) by restricting the initial probability of choosing each alternative to $\frac{1}{2}$, and assuming that the individual's revision rules are the same for all alternatives. The interpretation of $\frac{1}{2}$ in (1), however, is slightly different: it corresponds to the statistician's prior that the alternative chosen by the individual in the first period is the one with the dominant distribution; while subtle, this difference is critical for our experimental analysis, as it allows us to impose the statistician's prior in the design of the experiment. By contrast, in the traditional adaptive-learning setup, the probability with which an individual chooses an alternative in the first period reflects its "attraction" to the alternative; hence, it cannot be observed by the statistician or be controlled by the experimental design. Despite this subtle difference, the isomorphism of the computations allows us to obtain Propositions 1 and 3 in a way that is similar to how Oyarzun and Sarin (2013) obtain their Propositions 1 and 2 in the traditional setup.

[20] For example, when a parent chooses among the same two schools for each of his two children (sequentially), if he has an increasing response function then when it comes time to make the choice for his second child, it is more likely that he chooses the better school.

Corollary 1 *There is no response function θ such that*

$$\mathbb{E}P_a(\theta) \geq \mathbb{E}P_a(\theta')$$

for all response functions θ', and for all pair of distributions F_a and $F_{a'}$ in which F_a first-order stochastically dominates $F_{a'}$.

Corollary 1 asserts that no response function yields the best performance for all possible pairs of payoff distributions. Therefore, Propositions 1 and 2, together with Corollary 1, give us a partial idea of what we might expect of individuals' response functions if they were to be shaped by the performance criteria considered in these results. In particular, we might expect increasing response functions that are of maximum strength, as this would make choice of first-order stochastically dominant distributions more probable (if distributions can be ranked according to this criterion).

Curvature. Aside from its implications for the increasingness of a response function, Proposition 1 also establishes that, when payoff distributions can be ordered according to second-order stochastic dominance, response functions that are concave choose the alternative with less payoff variance more often in the second period. While we observe increasing response functions in our statistical representation of the data (see Section 4), we do not observe concave (or convex) response functions. We do, however, observe the following weaker notion of concavity:

Definition 1 *A response function θ is* point-concave (point-convex, point-linear) *at $x \in X$ if*

$$\theta(x) > (<, =)\frac{1}{2}\theta(x') + \frac{1}{2}\theta(x'')$$

for all $x', x'' \in X \setminus \{x\}$ such that $\frac{x'+x''}{2} = x$.

Standard computations then reveal the following:

Observation 1 *Suppose that $F_{a'}$ is constructed by applying a mean-preserving spread to F_a, which spreads probability mass from x to x' and x'', in equal amounts. Then, $\mathbb{E}P_a(\theta) > (<, =)\frac{1}{2}$ if and only if θ is point-concave (point-convex, point-linear) at x.*

This observation says that if individuals have response functions that are point-concave (point-convex) at x, then they will be more likely to pick the second-order stochastically dominant (dominated) distribution in the second period, where the dominated distribution is obtained by performing a symmetric mean-preserving spread on the dominant distribution, around x.

General distributions. We now relax the assumption that payoff distributions are ordered in terms of stochastic dominance. Absent a first-order stochastic dominance ordering across payoff distributions it is no longer clear: (i) that an increasing response function tends to lead to better payoffs, and (ii) how to even evaluate the performance of second period choices. In order to address (ii) we order the payoff distributions according to the natural criterion of which an Expected Utility maximizer would prefer. With this, we address (i) via the following result, in which $U(a)$ is the expected utility corresponding to payoff distribution $F_a(x)$, i.e., $U(a) = \int_X u(x)F_a(x)$, where $u : X \to \mathbb{R}$ is a strictly increasing Bernoulli utility function.

Proposition 3 *Consider an Expected Utility maximizer with strictly increasing Bernoulli utility function u and any two payoff distributions F_a and $F_{a'}$. If she prefers F_a to $F_{a'}$ and her response function is $\theta = b + cu$, for some real constants b and $c > 0$, then (i) $\mathbb{E}P_a(\theta) \geq \mathbb{E}P_{a'}(\theta)$ and, equivalently, (ii)*

$$\mathbb{E}P_a(\theta)U(a) + \mathbb{E}P_{a'}(\theta)U(a') \geq \frac{1}{2}U(a) + \frac{1}{2}U(a').$$

The first part of Proposition 3 establishes that an Expected Utility maximizer can expect to choose the (unknown) payoff distribution that she would prefer (if distributions were known) in the second period more often than the less-preferred distribution, if her response function is an affine transformation of her increasing Bernoulli utility function. The second part says that her ex-ante expected utility from the second period payoff is higher with such a response function than it would be by simply choosing alternatives at random (uniformly).

Lastly, the following corollary highlights that the relative benefits of an affine transformation of the increasing Bernoulli utility function are maximized by the unique affine transformation that stretches the response function to maximum strength.

Corollary 2 *Consider an Expected Utility maximizer with strictly increasing Bernoulli utility function u and any pair of payoff distributions F_a and $F_{a'}$ such that she prefers F_a to $F_{a'}$. Let $b^* = -u(\underline{x})\left(u(\overline{x}) - u(\underline{x})\right)^{-1}$, $c^* = \left(u(\overline{x}) - u(\underline{x})\right)^{-1}$, and $\theta^* = b^* + c^*u$. Then $\mathbb{E}P_a(\theta^*) - \mathbb{E}P_{a'}(\theta^*) \geq \mathbb{E}P_a(\theta) - \mathbb{E}P_{a'}(\theta)$ for all response functions $\theta = b + cu$ (where b and c are real constants).*

2.2 Response functions induced by Bayesian decision-making

Towards the end of Subsection 2.1, in a completely belief-free analysis, we explore how the preferences of an Expected Utility maximizer can be used to advantageously shape her response function. By contrast, here we explore what type of response functions would result from an Expected Utility maximizer who (i) holds prior beliefs about the payoff distributions that are possible for each alternative, (ii) upon experiencing a first period payoff updates beliefs according to Bayes rule, and (iii) uses updated beliefs to choose an alternative in the second period. The analysis contained in the remainder of this section is not meant to introduce deep theoretical developments. Instead it aims to provide the implications of standard decision models for the shape of response functions.

The set of all possible pairs of distributions on X that are considered in the individual's beliefs is denoted by E, and ξ is a sigma-algebra of E's subsets. A belief is a probability measure, $\mu : \xi \to [0,1]$, mapping every set in ξ to the (subjective) probability that the pair of payoff distributions of the chosen and unchosen alternatives is contained in that set. The distributions of the chosen and unchosen alternatives are denoted by G_a and G_{a^-}, respectively. Let $g_a(x)$ be the density of G_a at $x \in X$ if G_a is absolutely continuous; if G_a is a discrete distribution, then let $g_a(x)$ be the probability of obtaining x when drawing from the distribution G_a.[21] We assume that $\int_E g_a(x)d\mu > 0$ for all $x \in X$. Thus, upon choosing an alternative and obtaining a payoff of x, the individual can use Bayes' rule to compute her updated subjective beliefs that the pair of distributions (G_a, G_{a^-}) is in the set S; in particular, we define the updated beliefs $\mu_x(S) := \frac{\int_S g_a(x)d\mu}{\int_E g_a(x)d\mu}$, for all $S \in \xi$. Given these updated beliefs, the expected utilities of staying, and switching, respectively, are given by:

$$U_{\text{stay}}(x) = \int_E U(a)d\mu_x = \frac{\int_E g_a(x)U(a)d\mu}{\int_E g_a(x)d\mu} \quad \text{and} \quad U_{\text{switch}}(x) = \int_E U(a^-)d\mu_x = \frac{\int_E g_a(x)U(a^-)d\mu}{\int_E g_a(x)d\mu},$$

where $U(a) := \int_X u(z)dG_a(z)$ and $U(a^-) := \int_X u(z)dG_{a^-}(z)$, and $u : X \to \mathbb{R}$ is the individual's Bernoulli utility function.

Definition 2 *Given beliefs μ and Bernoulli utility function u, the* conditional expected difference *between the utility of staying vs. switching is a function $\rho : X \to \mathbb{R}$, with $\rho(x) := U_{stay}(x) - U_{switch}(x) = \int_E (U(a) - U(a^-))\, d\mu_x$, or equivalently:*

$$\rho(x) = \frac{\int_E g_a(x)\,(U(a) - U(a^-))\,d\mu}{\int_E g_a(x)d\mu}$$

for all $x \in X$, where x is the obtained first period payoff.

We thus have the following characterization of the response function of a Bayesian Expected Utility maximizer:

Observation 2 *The response function θ_B of a Bayesian Expected Utility maximizer with prior beliefs μ and Bernoulli utility function u, is*

$$\theta_B(x) \begin{cases} = 0 & \text{if } \rho(x) < 0 \\ \in [0,1] & \text{if } \rho(x) = 0 \\ = 1 & \text{if } \rho(x) > 0. \end{cases}$$

[21] Mixed distributions (see, e.g., DeGroot (2005)) can be handled in a fairly similar way.

In particular, if ρ is continuous and single crossing,[22] *then $\theta_B(x) = 0(\in [0,1], = 1)$ for $x < (=, >)s$, with s being defined by $\rho(s) \equiv 0$.*

Thus, a Bayesian Expected Utility maximizer has a response function that is extreme in the sense of staying with the same alternative in the second period with probability one, or switching with probability one, depending on the first period payoff.[23] In Appendix B, we provide several examples that illustrate: (i) how response functions are obtained within this framework, (ii) simple belief structures that yield response functions that are determined by a first-period payoff cut-off such that the individual switches (stays) for payoffs smaller (larger) than the cut-off, and (iii) beliefs that yield non-monotone response functions.

2.3 Random decision-making

We extend the analysis of Subsection 2.2 by allowing an individual's decision-making to be affected by a stochastic component. In particular, we consider a *random parameter* model (c.f., Apesteguia and Ballester (2017)), in which an underlying behavioral parameter of the individual, e.g. risk preferences, is assumed to be stochastic. Under this approach, we derive the response function, characterizing its slope for all $x \in X$.

We assume that individual preferences over staying and switching upon experiencing a payoff x in the first round are represented by the functions $U_{\text{stay}}, U_{\text{switch}} : X \times \Theta \to \mathbb{R}$, such that the individual prefers to stay (switch) upon experiencing x, if $U_{\text{stay}}(x, \omega) > (<)\, U_{\text{switch}}(x, \omega)$, where ω may be a vector of random parameters.[24] For simplicity, we further assume: (i) Θ is an interval of real numbers, (ii) staying and switching are Θ-ordered (cf., Apesteguia and Ballester (2017)), i.e., $U_{\text{stay}}(x, \omega) > U_{\text{switch}}(x, \omega)$ and $\omega' > \omega$ imply $U_{\text{stay}}(x, \omega') > U_{\text{switch}}(x, \omega')$, for all $x \in X$, and (iii) the distribution of ω, denoted by F_ω, is differentiable with density $f_\omega > 0$. Under these assumptions,

$$\theta_P(x) = 1 - F_\omega(\omega^*(x)), \tag{3}$$

where $\omega^*(x) := \sup\{\omega \in \Theta : U_{\text{stay}}(x, \omega) < U_{\text{switch}}(x, \omega)\}$ is the cut-off value of the parameter, above (below) which the individual stays (switches). If we let $U_{\text{stay}}(x, \omega)$ and $U_{\text{switch}}(x, \omega)$ be continuously differentiable over $X \times \Theta$, and $U_{\text{stay}}(x, \omega) - U_{\text{switch}}(x, \omega)$ have a strictly positive partial derivative with respect to ω, then (by the Implicit Function Theorem) $\omega^*(x)$ is defined implicitly by $U_{\text{stay}}(x, \omega^*(x)) \equiv U_{\text{switch}}(x, \omega^*(x))$ and

$$\frac{d\theta_P(x)}{dx} = -f_\omega\left(\omega^*(x)\right) \frac{d\omega^*(x)}{dx}, \tag{4}$$

[22] By ρ being single crossing, we mean that there is $s \in X$ such that $\rho(x) < (=, >) 0$ for $x < (=, >) s$.

[23] This is true up to the payoff, or payoffs, at which he is indifferent between staying, switching, and any probability-mixture of the two.

[24] While this specification for $U_{\text{stay}}(x, \omega)$ and $U_{\text{switch}}(x, \omega)$ admits, as a particular case, the Bayesian Expected Utility maximizer considered before, with ω being a parameter perhaps defining the shape u, it also allows other possibilities, including non-expected utility preferences or other behavioral traits (see, e.g., Example 3 below).

for all $x \in X$.[25]

The interpretation of $\frac{d\theta_P(x)}{dx}$ is direct: it corresponds to the product of the rate of change of the cut-off value of the random parameter with respect to the experienced payoff, times the rate of change of the probability of the set of parameter values above the cut-off (given by the additive inverse of the density of the random parameter).

Appendix C provides (i) an example of a random parameter model in which the decision-maker errs when performing Bayesian updating, and (ii) analogous derivations to those provided in this subsection, for the *random utility* approach (e.g., Hey and Orme (1994), Loomes and Sugden (1995)), under which decision-makers (now with deterministic behavioral parameters) sometimes make an error when choosing between alternatives.[26] Appendix D shows how to accommodate ambiguity aversion within the setup of Subsections 2.2 and 2.3.

3 Experimental design

The experiment was conducted at the University of Alicante's LaTEx laboratory. A university-wide pool of 566 undergraduates registered to participate in "paid decision-making experiments," responding to fliers placed around campus, and email invitations. Of the 566, 100 were selected at random to participate in the experiment.

The duration of each session was approximately 45 minutes. Subjects were lead into the laboratory and seated, at which point instructions were read aloud by the experimenter.[27] Following the instructions subjects performed three "practice" decision tasks, then took a short, incentivized, test of their understanding of the instructions. Following the test subjects were permitted to ask the experimenter any questions they had before starting the 40 paid tasks.[28] Upon completing the tasks subjects filled out a brief questionnaire, before being paid, and then left the laboratory.

Each decision task consists of two rounds. In the first round, one of two "alternatives" must be selected. Each alternative is represented by a generic rectangle on the computer screen—one is located on the left side, at middle height, and the other is on the right side, at middle height. Aside from their spatial positions the rectangles are visually identical. One of the two rectangles must be selected by clicking on it with the mouse. Once selected, the rectangle disappears, and in its place appears a number—the subject's first round "payoff," in Euros. The subject then clicks to proceed to the second round, at which point the payoff disappears and the original two generic rectangles reappear. In the second round the subject must again select one of the two

[25]The derivative $\frac{d\omega^*(x)}{dx}$ can be obtained using the Implicit Function Theorem.

[26]An important caveat is that Wilcox (2008), Wilcox (2011), and Apesteguia and Ballester (2017) have recently shown that the random utility approach is problematic in many important cases, especially for the purposes of empirically estimating utility functions. The random parameter approach is not vulnerable to these issues (Apesteguia and Ballester (2017)) and neither is our empirical analysis, as our estimation of response functions is altogether free of utility functions.

[27]See Appendix E for the instructions.

[28]The experimental software used was **z-Tree** (see Fischbacher (2007)).

rectangles, at which point the rectangle again disappears, but this time no payoff appears. The subject then clicks to move on to the next task, and so on.

The subject knows that once she has completed the 40 decision tasks, one will be selected at random in order to determine her payoffs. The payoff of the first round plus twice the payoff of the second round of the selected task will be paid to her.[29]

While subjects know the payoff mechanism, they know very little about how the payoffs of each alternative in each task are generated. Subjects *do* know the following about the alternatives: within each task, each alternative generates payoff(s) according to a well-defined probability distribution. The support is known to be $\{1, 2, 3, 4, 5\}$ across the whole experiment. Draws from the same alternative in the same task yield independent payoff realizations (i.e., are made "with replacement"). While alternatives are stationary within a task, as each alternative distribution is the same in both rounds, these distributions change from one task to the next. Furthermore, subjects are not told anything about the distribution of probabilities over payoffs. The objective of the design is thus to elicit subjects' "primitive" responses to experienced payoffs, in a context in which they initially have no information about what distribution is generating the payoffs. We want to know simply whether experiencing a payoff of x Euros makes someone want to *stay* with the same alternative, or *switch*.[30]

While subjects are told nothing about how probabilities are distributed across payoffs for each alternative, the pairs of alternatives in each task have a clear dominance relationship. Table 1 shows pairs of alternatives in adjacent rows labeled according to the dominance relation. In 20 decision tasks, one alternative first-order stochastically dominates the other. In 10 of these tasks, the dominance relation is "Strong FOSD" in the sense that the first-order stochastically dominant distribution assigns virtually all the probability mass to the payoff 5, whereas the dominated distribution assigns virtually all the probability mass to the payoff 1. For the other 10 of these decision tasks, labeled "Weak FOSD," the dominance relation is slightly weaker in the sense that the dominant distribution assigns virtually all the probability mass to the payoff 4, whereas the dominated distribution assigns virtually all the probability mass to the payoff 2. In the remaining 20 tasks, one alternative second-order stochastically dominates the other. In 10 of them, labeled "Strong SOSD," the dominant distribution assigns virtually all of the probability mass to the payoff 3, whereas the dominated distribution spreads virtually all of the probability mass between the payoffs 1 and 5. In the other 10 decision tasks, labeled "Weak SOSD," the dominant distribution assigns virtually all of the probability mass to the payoff 3, whereas the dominated distribution spreads virtually all of the probability mass between the payoffs 2 and 4.

[29]Subjects could earn a minimum of 3 and a maximum of 16 Euros. On average they earned 10.66 Euros, which included earnings of 25 cents for each correct response to the four comprehension test questions.

[30]We go to great lengths in the instructions, as well as the accompanying paid comprehension test, to ensure that subjects understand that 1) each task should be treated independently, 2) payoffs are random variables, and 3) payoff distributions for each alternative are stationary (with replacement) across rounds of the same task. For details, see Appendix E.

Table 1: Payoff distributions used in decision tasks.

	Dominant Distribution					Dominated Distribution				
	1	2	3	4	5	1	2	3	4	5
Strong FOSD	0.01	0.01	0.01	0.01	0.96	0.96	0.01	0.01	0.01	0.01
Weak FOSD	0.01	0.01	0.01	0.96	0.01	0.01	0.96	0.01	0.01	0.01
Strong SOSD	0.01	0.01	0.96	0.01	0.01	0.47	0.02	0.02	0.02	0.47
Weak SOSD	0.01	0.01	0.96	0.01	0.01	0.02	0.47	0.02	0.47	0.02

Each pair of alternatives appeared in 10 tasks, with the dominant alternative on the left hand side of the screen half of the time, and on the right hand side of the screen the other half. The order of the 40 tasks was randomized. Therefore, the payoff distribution for either alternative *across* tasks was the same (up to re-ordering), and within each alternative the same for "low" or "high" payoffs, i.e. identical for payoffs 1,2,4, and 5, with average probabilities near 0.20 for each (0.1875).[31] Thus, the elicitation of each subject's response function was not sensitive to the specific relationships existing between the payoff distributions, as it was not possible for subjects to infer what these relationships were. This design feature allows us to consider the response functions we elicit as primitives, whereby they can be used to perform the counterfactual exercise of exploring the payoff consequences of subjects' choice behavior in an environment characterized by any pair of (unknown) payoff distributions.

While many different combinations of distributions could have worked in order to elicit subjects' response functions, by using distributions that can be ordered according to stochastic dominance we have the advantage of being able to empirically illustrate (see Section 5) the theoretical predictions reported in Section 2 (with effect sizes).

4 Estimating response function classes

We estimate a mixture model that allows for the identification of different classes of response functions across subjects. While mixture models have a long tradition in statistics (see, e.g., McLachlan and Peel (2000)), they are not used as often in experimental economics to study the heterogeneity across subjects' behavior.[32] In this section we describe our mixture model and its corresponding estimation results. The underlying model of our empirical analysis is a standard finite mixture of logistic regressions, see e.g., McLachlan and Peel (2000), Section 5.10. The identifiability of such models has long been known in the neural networks literature (cf. Jiang and Tanner (1999)).

[31]The average probability of the middle payoff 3 was slightly higher (0.25) due to our objective of having a balanced number of decision tasks that were ordered according to first- and second-order stochastic dominance, respectively.

[32]One exception is El-Gamal and Grether (1995), who study different classes of Bayesian updating, and deviations from Bayesian updating.

4.1 Mixture model

The model describes the probability that an individual repeats her first round choice in the second round, as a Bernoulli distribution whose underlying probability depends only on the payoff experienced in the first round. We assume that there are a number of different classes of individuals (we find 5 classes in the data). For each given first-round payoff, all individuals within a given class have the same probability of choosing the same alternative again in the second round. Each individual belongs to each class according to a multinomial distribution.

Classes. There are N classes of individuals, and a priori each individual belongs to Class $i \in \{1, 2, ..., N\}$ with probability $\pi_i \geq 0$, where $\sum_{i=1}^{N} \pi_i = 1$. The probability that an individual in Class i chooses the same alternative in both rounds, given that she experienced the payoff $k \in X := \{1, ..., 5\}$ in the first round, is

$$\theta_{i,k} := \frac{e^{\alpha_{i,k}}}{1 + e^{\alpha_{i,k}}}$$

for all $i \in \{1, 2, ..., N\}$, where $\alpha_{i,k} \in \mathbb{R}$ is a parameter. Let $x_{j,t} \in X$ be the payoff experienced by individual j in the first round of task t for $j \in J := \{1, 2, ..., 100\}$ and $t \in T := \{1, 2, ..., 40\}$. It will be convenient to write the probability that the individual j in the Class i chooses the same alternative in the two rounds of the decision task t, given that she obtained a payoff $x_{j,t}$ in the first round, as

$$p_{i,j,t} := \frac{\exp\left(\boldsymbol{x}_{j,t} \cdot \boldsymbol{\alpha}_i\right)}{1 + \exp\left(\boldsymbol{x}_{j,t} \cdot \boldsymbol{\alpha}_i\right)}, \tag{5}$$

where $\boldsymbol{\alpha}_i = (\alpha_{i,1}, ..., \alpha_{i,5})'$ is a vector containing the parameters $\alpha_{i,k}$ with $k \in X$ and $\boldsymbol{x}_{j,t} = (x_{j,t,1}, ..., x_{j,t,5})'$, where $x_{j,t,k} = 1$ if $x_{j,t} = k$ and $x_{j,t,k} = 0$, otherwise.

The likelihood function. Let $y_{j,t} \in \{0, 1\}$ indicate whether individual j chose the same alternative in both rounds of the decision task t, $y_{j,t} = 1$, or if she decided to switch, $y_{j,t} = 0$, for $j \in J$ and $t \in T$. Let $\Psi := \left(\pi_i, (\alpha_{i,k})_{k=1}^{5}\right)_{i=1}^{N}$ be the set of parameters. The subjects' first round payoffs are represented by the three-dimensional array $\mathbf{X} := (\boldsymbol{x}_{j,t})_{t\in T, j\in J}$, and their second round choices are represented by the matrix $\mathbf{Y} := \left(y_{j,t}\right)_{j\in J, t\in T}$. Then, we can write the likelihood function as[33]

$$L\left(\Psi; \mathbf{Y}, \mathbf{X}\right) := \prod_{j=1}^{100} \left(\sum_{i=1}^{N} \pi_i \left(\prod_{t=1}^{40} p_{i,j,t}^{y_{j,t}} \left(1 - p_{i,j,t}\right)^{1-y_{j,t}} \right) \right).$$

Complete-data likelihood function. The Expectation-Maximization (EM) algorithm that we use (see,

[33] Our model is identified up to a permutation (see, e.g., Section 3.1 in Titterington et al. (1985)); inference based on the estimates provided below can be justified by the extreme estimator consistency theorem (Theorem 4.1.2 in Amemiya (1985)).

e.g., Dempster et al. (1977) and McLachlan and Krishnan (2008)) requires us to introduce an additional unobservable random vector $\boldsymbol{z}_j = (z_{1,j}, ..., z_{N,j})'$, with $z_{i,j} = 1$ if individual j belongs to Class i and $z_{i,j} = 0$, otherwise. By construction of the model, $\boldsymbol{z}_j$ has a multinomial distribution with parameters $\boldsymbol{\pi} = (\pi_1, ..., \pi_N)'$, for all $j \in J$. If $\boldsymbol{z}_j$ were observable for all j, we could write the *complete-data likelihood function,*

$$L_c(\Psi; \mathbf{Y}, \mathbf{X}) := \prod_{j=1}^{100} \left(\prod_{i=1}^{N} \left(\pi_i \left(\prod_{t=1}^{40} p_{i,j,t}^{y_{j,t}} (1 - p_{i,j,t})^{1-y_{j,t}} \right) \right)^{z_{i,j}} \right)$$

and the corresponding complete-data log-likelihood function

$$l_c(\Psi; \mathbf{Y}, \mathbf{X}) := \sum_{j=1}^{100} \sum_{i=1}^{N} z_{i,j} \left(\log(\pi_i) + \sum_{t=1}^{40} (y_{j,t} \log(p_{i,j,t}) + (1 - y_{j,t}) \log(1 - p_{i,j,t})) \right).$$

The algorithm estimates parameters by iterating through the Expectation (E-) and Maximization (M-) steps until convergence, as described in Appendix F. In a nutshell, in the E-step we compute the expected value of the complete-data log-likelihood function, replacing the $z_{i,j}$ terms by their conditional expected values. These expected values are computed with the parameters estimated in the previous iteration $\Psi^{(-1)}$, i.e., replacing the $z_{i,j}$ terms by its conditional expected value:

$$\begin{aligned} \tau_{i,j}^{(-1)} &:= \mathbb{E}_{\Psi^{(-1)}} [z_{i,j} | \mathbf{Y}, \mathbf{X}] \\ &= \mathbb{P}_{\Psi^{(-1)}} (z_{i,j} = 1 | \mathbf{Y}, \mathbf{X}) \\ &= \frac{\pi_i \left(\prod_{t=1}^{40} \left(p_{i,j,t}^{(\Psi^{(-1)})} \right)^{y_{j,t}} \left(1 - p_{i,j,t}^{(\Psi^{(-1)})} \right)^{1-y_{j,t}} \right)}{\sum_{i'=1}^{N} \pi_{i'} \left(\prod_{t=1}^{40} \left(p_{i',j,t}^{(\Psi^{(-1)})} \right)^{y_{j,t}} \left(1 - p_{i',j,t}^{(\Psi^{(-1)})} \right)^{1-y_{j,t}} \right)}, \end{aligned}$$

where $\mathbb{P}_{\Psi^{(-1)}} (z_{i,j} = 1 | \mathbf{Y}, \mathbf{X})$ is the probability of $z_{i,j} = 1$, given $\mathbf{Y}, \mathbf{X}$, and $p_{i,j,t}^{(\Psi^{(-1)})}$ is computed with the parameters from the previous iteration. Once we have computed $\mathbb{E}_{\Psi^{(-1)}} [l_c(\Psi; \mathbf{Y}, \mathbf{X})]$, we proceed to the M-step, i.e., maximizing $\mathbb{E}_{\Psi^{(-1)}} [l_c(\Psi; \mathbf{Y}, \mathbf{X})]$ with respect to Ψ. Then, the algorithm iterates until convergence. The maximum likelihood estimates are the final iterates of the algorithm and are denoted by $\hat{\Psi} := \left(\hat{\pi}_i, (\hat{\alpha}_{i,k})_{k=1}^{5} \right)_{i=1}^{N}$.

4.2 Estimated classes

We estimate the parameters $\left(\pi_i, (\alpha_{i,k})_{k=1}^{5} \right)_{i=1}^{N}$ for $N = 1, 2, ...$ With the estimates we can compute the Bayesian information criterion (BIC) for each possible number of classes and select the number N that yields the minimum BIC. The BIC estimations are provided in Figure 1. Following this criterion, the sample of subjects is split into

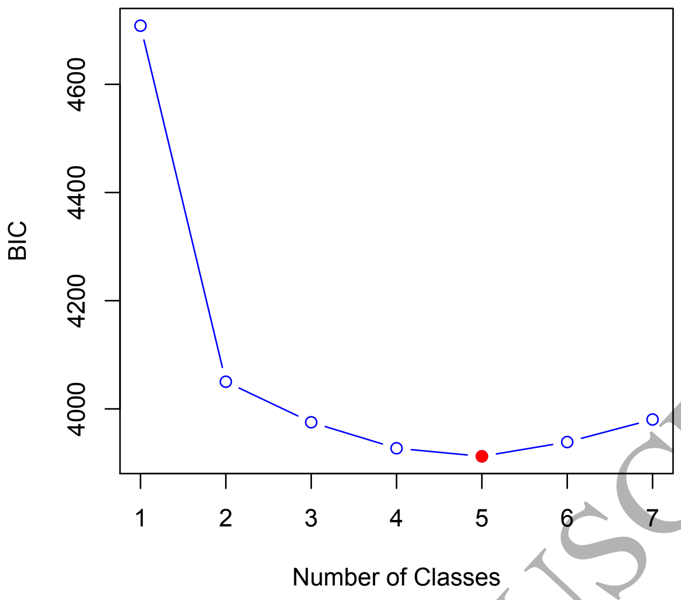

Figure 1: Number of classes and Bayesian information criterion.

five classes.[34] The parameter estimates for the five classes $\left(\hat{\theta}_{i,k}\right)_{i,k=1}^{5}$, and their 99% confidence intervals, are provided in Table 2.[35] This table also provides the estimates $(\hat{\pi}_i)_{i=1}^{5}$ that correspond to the prior probabilities that a subject belongs to each class. Figure 2 provides a plot of the estimated response functions, along with a 99% confidence interval, for all five classes.

Table 2: Parameter estimates of the model and 99% confidence intervals.

	$\hat{\theta}_{i,1}$	$\hat{\theta}_{i,2}$	$\hat{\theta}_{i,3}$	$\hat{\theta}_{i,4}$	$\hat{\theta}_{i,5}$	$\hat{\pi}_i$
Class 1	0.00 $(-,-)$*	0.01 $(0.00, 0.08)$	0.94 $(0.86, 0.97)$	1.00 $(0.89, 1.00)$	0.99 $(0.89, 1.00)$	0.21 $(0.00, 0.42)$
Class 2	0.05 $(0.02, 0.12)$	0.08 $(0.04, 0.16)$	0.47 $(0.38, 0.55)$	0.95 $(0.89, 0.98)$	0.96 $(0.89, 0.98)$	0.27 $(0.03, 0.51)$
Class 3	0.35 $(0.24, 0.49)$	0.49 $(0.34, 0.63)$	0.84 $(0.73, 0.91)$	0.92 $(0.81, 0.97)$	0.95 $(0.81, 0.99)$	0.14 $(0.00, 0.29)$
Class 4	0.36 $(0.26, 0.47)$	0.35 $(0.25, 0.47)$	0.40 $(0.30, 0.50)$	0.47 $(0.37, 0.59)$	0.61 $(0.50, 0.71)$	0.21 $(0.10, 0.32)$
Class 5	0.75 $(0.63, 0.84)$	0.71 $(0.57, 0.82)$	0.60 $(0.49, 0.70)$	0.48 $(0.37, 0.59)$	0.40 $(0.28, 0.52)$	0.17 $(0.00, 0.37)$

*As $\hat{\theta}_{1,1}$ approaches 0, the numerical algorithm of **flexmix R** becomes unstable and fails to provide the standard error. For practical purposes, in the sequel the confidence interval of $\hat{\theta}_{1,1}$ is approximated to have lower and upper bound 0; all individuals assigned to Class 1 (see Subsection 4.3) always switch after a first round payoff of 1.

[34] We estimated the model for $N = 1, 2, ..., 7$. Models with $N > 7$ classes were also considered, but the EM algorithm failed to converge to plausible parameter values.

[35] The estimation has been done with the the **flexmix R** package (see Appendix F).

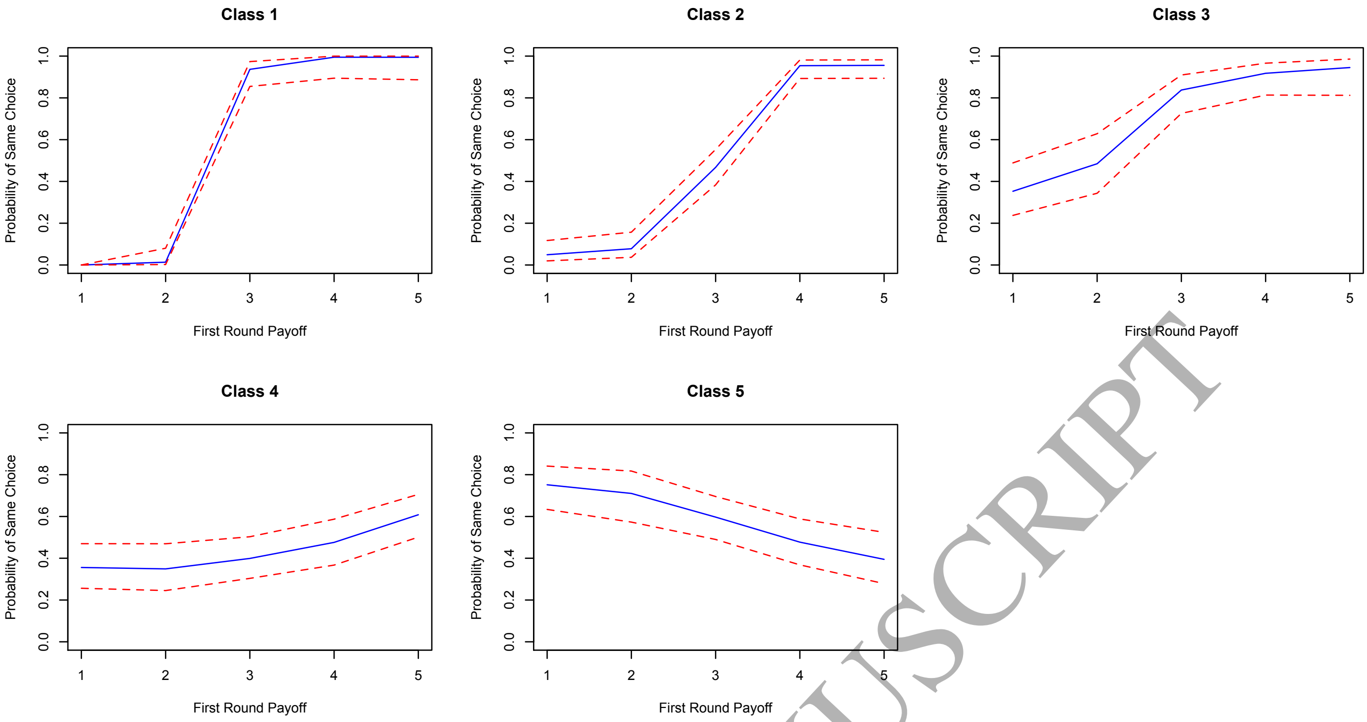

Figure 2: Response function estimates with 99% confidence interval.

In Figure 2, the 99% confidence interval of the response function estimates for Classes 1-4 reveal that these functions are all *statistically increasing*, i.e., for any x and x' such that the confidence intervals do not intersect, $x' > x$ implies $\hat{\theta}_{i,x'} > \hat{\theta}_{i,x}$, and not all intervals intersect.[36] In particular, two strongly responsive classes of subjects (Classes 1 and 2) respond positively, and somewhat dramatically, to payoffs. When individuals in these classes experience a first round payoff of less than 3, the probability of choosing the same alternative in the second round is virtually zero, whereas for experienced payoffs greater than 3, the probability of choosing the same alternative in the second round is virtually one. These individuals display behavior that is very close to what is prescribed by maximum strength. This is especially true of Class 1, where all individuals always switched upon obtaining a payoff of 1, and in all but a single choice by one individual, they chose to stay when the payoff received was a 5 (for details on how individuals were assigned to each class, see Subsection 4.3).

What distinguishes Classes 1 and 2 is the second round decisions that immediately follow an experienced first round payoff of 3. Whereas subjects in Class 1 choose the same alternative in the second round with probability very close to 1, subjects in Class 2 choose the same alternative only half of the time. More precisely, the response function of individuals in Class 1 is *statistically point-concave at* $x = 3$, i.e., the confidence interval of $\theta_{1,3}$ is above the equally weighted convex combination of the upper bounds of the confidence intervals of $\theta_{1,2}$ and $\theta_{1,4}$ and above the equally weighted convex combination of the upper bounds of the confidence intervals

[36] Analogously, we say that a response function is *statistically decreasing* if for any x and x' such that the intervals do not intersect, $x' > x$ implies $\hat{\theta}_{i,x'} < \hat{\theta}_{i,x}$, and not all intervals intersect.

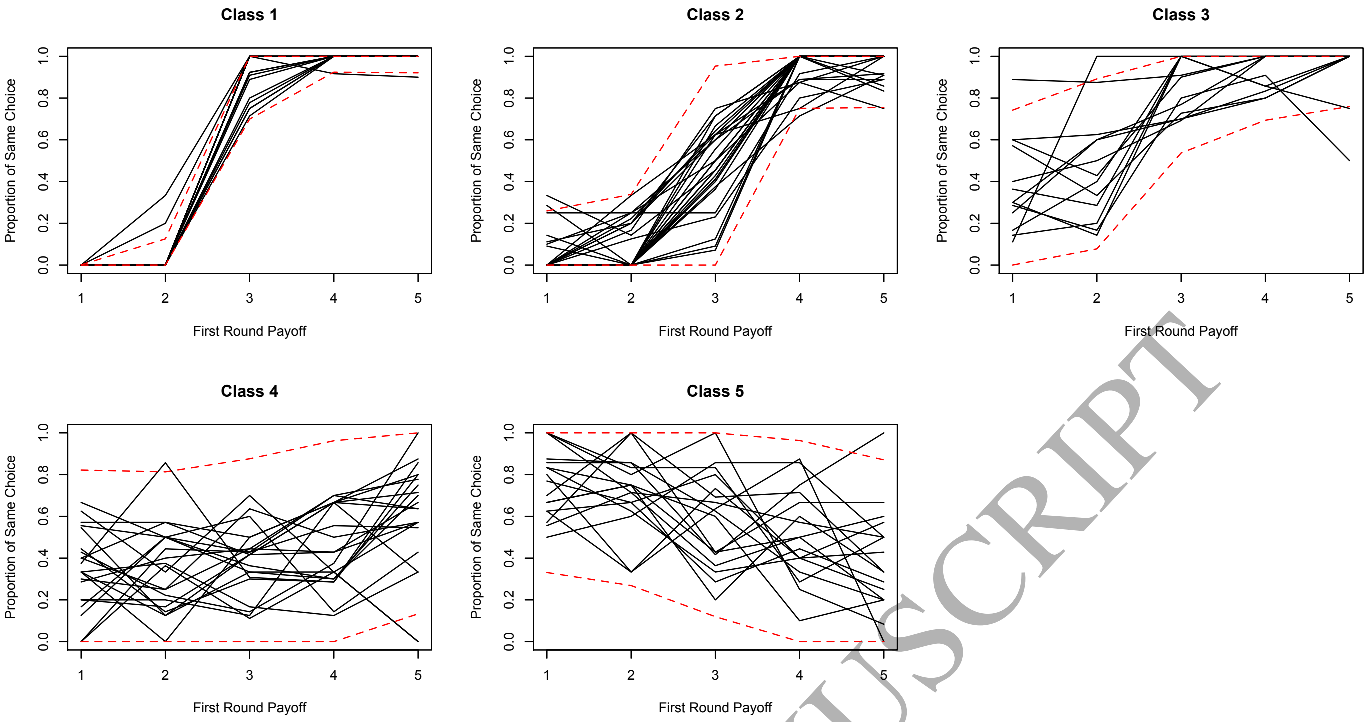

Figure 3: Actual frequencies and 99% confidence interval for proportions of same choice.

of $\theta_{1,1}$ and $\theta_{1,5}$.[37] The estimates reveal that Classes 1 and 2 represent 21% and 27% of the subjects from the sample, respectively.

A third class of subjects, Class 3, is similar to the two classes previously described in that the probability of choosing the same alternative in both rounds of a decision task is an increasing function of the payoff experienced in the first round. The main difference between Class 3 and Classes 1 and 2 can be seen in Table 2: the probability of choosing the same alternative in the second round, after experiencing a (low) payoff of 1 or 2 in the first, for Class 3 is at least 1/3, whereas for Classes 1 and 2 it is very close to zero. Hence, the response function of Class 3 clearly violates maximum strength. This class represents 14% of subjects. Finally, for the two remaining classes, the probability of choosing the same alternative in both rounds of a decision task, for any two different experienced payoffs, never differs by more than around 1/3. Whereas subjects in Class 4 exhibit a response function that is statistically increasing, the response function of subjects in Class 5 is statistically decreasing. These two classes represent 21% and 17% of the sample, respectively.

[37]On the other hand, the response function of individuals in Class 2 is *statistically point-linear at* $x = 3$; i.e., the confidence interval of $\theta_{2,3}$ intersects the interval obtained by the equally weighted convex combination of the upper (lower) bounds of the confidence intervals of $\theta_{2,2}$ and $\theta_{2,4}$, and the interval obtained by the equally weighted convex combination of upper (lower) bounds of the confidence intervals of $\theta_{2,1}$ and $\theta_{2,5}$.

4.3 Class assignation

For each subject j and class i, we can provide an a posteriori estimate of the probability that subject j belongs to class i,

$$\hat{\tau}_{i,j} \quad := \quad \frac{\hat{\pi}_i \left(\prod_{t=1}^{40} \left(\hat{p}_{i,j,t}\right)^{y_{j,t}} \left(1 - \hat{p}_{i,j,t}\right)^{1-y_{j,t}} \right)}{\sum_{i'=1}^{5} \hat{\pi}_{i'} \left(\prod_{t=1}^{40} \left(\hat{p}_{i',j,t}\right)^{y_{j,t}} \left(1 - \hat{p}_{i',j,t}\right)^{1-y_{j,t}} \right)},$$

with $\hat{p}_{i,j,t}$ given by the right hand side of (5) evaluated at $\hat{\Psi}$. Following McLachlan and Basford (1988), each subject $j \in J$ is assigned to the class $i = \arg\max_{i' \in \{1,2,3,4,5\}} \hat{\tau}_{i'j}$. For each class i, $\hat{\tau}_{i,j}$ averages more than 0.94 among the subjects assigned to that class, with a minimum of 0.63 for all assignations. Overall, this indicates that the classes distinguish subjects' response functions reasonably well. Figure 3 shows the relative frequencies with which each subject from each class chooses the same alternative in both rounds of a task, as a function of first round payoff. This figure also provides the 99% confidence interval for the relative frequencies of same choice. The model allows us to make fairly good predictions for subjects in Classes 1 and 2 when the experienced payoff is high (4 or 5) or low (1 or 2). For the other classes, predictions are less accurate, which may be expected given that their response functions lie relatively closer to one half.

5 Performance of response function classes

We report how second round payoffs differ with response function class. First, we observe that subjects with response functions that are more responsive to payoffs obtain higher average payoffs than less responsive subjects, as expected from Propositions 1 and 2. Second, we report the effects of response function curvature on the riskiness of second round payoff distributions, in accordance with Observation 1. The purpose of reporting these empirical results is mainly illustrative, as from the theory results provided in Section 2 we know that, for instance, individuals with response functions that are more responsive are expected to exhibit, on average, higher payoffs (when payoff distributions are ordered according to first-order stochastic dominance). Nevertheless, by reporting these results, we give a sense of the magnitude of payoff differences across those response function classes that we observe in our experiment.[38]

[38]Further, because our elicitation of response functions is insensitive to the particular (unknown) payoff distributions (see Section 3), one could also similarly plug in whatever pair of payoff distributions they like and compare the corresponding expected payoff performance across any two classes of response function.

5.1 Alternatives ordered according to first-order stochastic dominance

First we compare the strongly responsive subjects (Classes 1 and 2) with the weakly and non-responsive subjects (Classes 3, 4, and 5). Proposition 1 in Section 2 states that when alternatives can be (strictly) ranked according to first-order stochastic dominance, then responsive subjects are on average more likely to choose the dominant alternative in their second round choices; this implies that they should have a higher expected payoff in the second round than if their choices were simply random or non-responsive to payoffs. We report that this is indeed the case.

Let T_ρ be set of decision tasks with pair of payoff distributions $\rho \in \{sf, wf\}$, where sf and wf indicate Strong FOSD and Weak FOSD, respectively (see Table 1). Let $x_{j,t}^{(2)}$ be the payoff obtained in the second round of the decision task t by subject j. For each subject $j \in J$ we compute the average payoff she obtained in the second round in all decision tasks in T_ρ, $\bar{x}_{j,\rho} := \frac{\sum_{t \in T_\rho} x_{j,t}^{(2)}}{|T_\rho|}$. Then, we compare the average second round payoffs of subjects in the responsive classes with those of subjects in the weakly responsive and non-responsive classes. Let J_i be the set of subjects that are assigned to Class i. The mean of the average second round payoff of subjects in Class i and decision tasks in T_ρ, denoted by $\mu_{i,\rho}^{(\bar{x})}$, is estimated as $\hat{\mu}_{i,\rho}^{(\bar{x})} := \frac{\sum_{j \in J_i} \bar{x}_{j,\rho}}{|J_i|}$.

Table 3 provides the average second round payoff for individuals in each class, for decision tasks in which the payoff distributions are ordered according to strong and weak first-order stochastic dominance. As anticipated by Proposition 1, individuals in Classes 1 and 2 obtain higher average second round payoffs than individuals in the other classes. Furthermore, these differences are significant. Table 4 provides the difference in average payoffs between Classes 1 and 2 and the weakly and non responsive classes (3, 4, and 5) for all decision tasks with pair of payoff distributions Strong FOSD (i.e., $\hat{\mu}_{i,sf}^{(\bar{x})} - \hat{\mu}_{i',sf}^{(\bar{x})}$ for classes $i = 1, 2$ and $i' = 3, 4, 5$). Table 4 also reports the p-values from a permutation test of mean differences.[39] These results are consistent with the theoretical predictions from Section 2; differences between expected payoffs of the strongly responsive, and weakly and non-responsive classes, are positive and significant at the 1% level for all comparisons. The largest differences are found when comparing subjects from Classes 1 and 2 with those of Class 5; subjects in Classes 1 and 2 earn on average 4.69 and 4.65 Euros, respectively, whereas subjects in Class 5 earn just 2.41 Euros on average.[40]

Table 3: Average second round payoffs (in Euros) for decision tasks ordered according to FOSD.

	Class 1	Class 2	Class 3	Class 4	Class 5
Strong FOSD	4.69	4.65	4.05	3.57	2.41
Weak FOSD	3.89	3.76	3.46	3.06	2.85

[39]For a brief description of the permutation test we used, see Appendix 6.3.

[40]Recall that when subjects were paid, round two payoffs were doubled, so the actual differences in earnings produced from round two payoffs are twice those observed in Tables 3 and 4.

Table 4: Test of (Strong FOSD) hypotheses H_0: $\mu^{(\bar{x})}_{i,sf} - \mu^{(\bar{x})}_{i',sf} \leq 0$ for $i = 1, 2$ and $i' = 3, 4, 5$.

		Class 3	Class 4	Class 5
Class 1	Difference	0.63	1.12	2.28
	p-value	0.00	0.00	0.00
Class 2	Difference	0.59	1.08	2.24
	p-value	0.00	0.00	0.00

Analogously, Table 5 provides the difference in average payoffs between strongly responsive classes (1 and 2) and weakly and non-responsive classes (3, 4, and 5), for payoff distributions with a Weak FOSD relationship (i.e., $\hat{\mu}^{(\bar{x})}_{i,wf} - \hat{\mu}^{(\bar{x})}_{i',wf}$ for $i = 1, 2$ and $i' = 3, 4, 5$). These results are qualitatively similar to those under Strong FOSD; differences in average payoffs are positive and significant at the 1% level for all comparisons. Quantitatively, these differences are relatively smaller, with subjects in Classes 1 and 2 obtaining an average of 3.89 and 3.76 Euros, respectively, and subjects in Class 5 earning an average of 2.85 Euros.

Table 5: Test of (Weak FOSD) hypotheses H_0: $\mu^{(\bar{x})}_{i,wf} - \mu^{(\bar{x})}_{i',wf} \leq 0$ for $i = 1, 2$ and $i' = 3, 4, 5$.

		Class 3	Class 4	Class 5
Class 1	Difference	0.43	0.83	1.03
	p-value	0.00	0.00	0.00
Class 2	Difference	0.30	0.70	0.91
	p-value	0.00	0.00	0.00

Finally, we compare distributions of average second round payoffs across the 5 observed classes of response functions. Figure 4 shows the estimated densities, using Bernstein polynomials (see, e.g., Babu et al. (2002)), of the average second round payoffs across classes, for those tasks in which the payoff distributions are ranked according to first-order stochastic dominance. Consistent with the results above, densities of Classes 1 and 2 concentrate probability mass to the right of 3, both in the case of weak and strong dominance relations. The density of average payoffs for Class 3's weakly responsive subjects also concentrates density above 3, but with more density below 4 than Classes 1 and 2. Finally, the non-responsive classes concentrate even more density below 4 than the weakly responsive Class 3; Class 5 is extreme in that the majority of its density is below the 3 Euro payoff, which is consistent with this response function having a slightly negative slope.

5.2 Alternatives ordered according to second-order stochastic dominance

We now test whether the differing curvature of response functions for the Classes 1 and 2 affects the spread of payoffs in tasks for which payoff distributions can be (strictly) ranked according to second-order stochastic dominance. As observed above, the main difference between Class 1 and 2 subjects is that when the experienced payoff is 3, subjects in Class 1 choose the same alternative in both rounds with probability almost 1, whereas

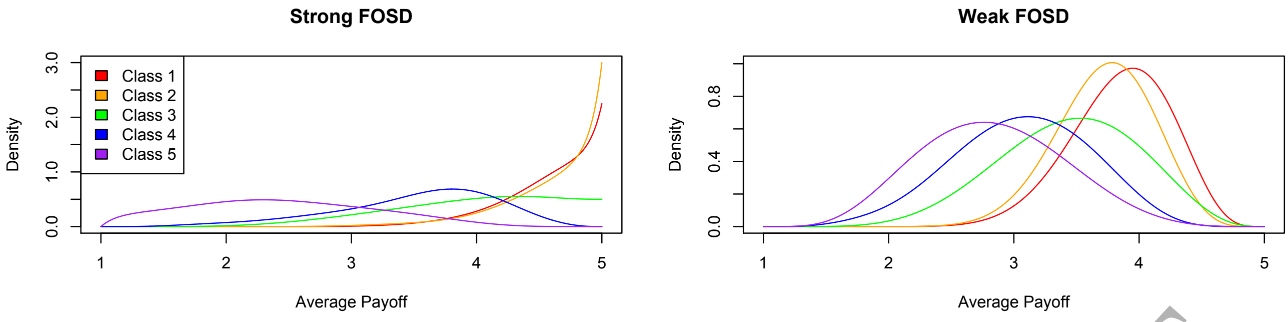

Figure 4: Second round payoff average densities, across subjects, estimated using Bernstein polynomials.

subjects in Class 2 do so only around half of the time. As a result the response function of subjects in Class 1 is point-concave at $x = 3$ and the response function of the subjects in Class 2 is statistically point-linear at $x = 3$. Therefore, Observation 1 in Section 2 predicts that subjects in Class 1 are, on average, more likely to choose second-order stochastic dominant alternatives in the second round of decision tasks with Strong FOSD and Weak FOSD pairs of payoff distributions; this also implies that they should be exposed to less risk in their second round payoffs.

For each subject $j \in J_1 \cup J_2$ we compute the second round payoff variance of all decision tasks in T_ρ, $s^2_{j,\rho} := \frac{\sum_{t \in T_\rho}\left(x^{(2)}_{j,t} - \bar{x}_{j,\rho}\right)^2}{|T_\rho|-1}$, with $\rho \in \{ss, ws\}$, where ss and ws indicate Strong SOSD and Weak SOSD, respectively. The mean of the second round payoff variance for subjects in Classes 1 and 2 in the decision tasks in T_ρ, denoted by $\mu^{(s^2)}_{i,\rho}$ with $i = 1, 2$, is estimated by $\hat{\mu}^{(s^2)}_{i,\rho} := \frac{\sum_{j \in J_i} s^2_{j,\rho}}{|J_i|}$ for $\rho \in \{ss, ws\}$. As above, we performed a permutation test to compare the average second round payoff variance of subjects in Class 1 with that of subjects in Class 2. The results are displayed in Table 6. As predicted by the theory, subjects in Class 1 have smaller average payoff variance than subjects in Class 2. The difference in this average exposure to risk between Class 1 and Class 2 subjects is 1.02 Euro^2 for those decision tasks in which the payoff distributions correspond to Strong SOSD.[41] For those decision tasks in which the dominance relation is weak, the difference in the average second round payoff variances is 0.20. Both of these differences are significant at the 5% level.

Table 6: Test of the hypotheses H_0: $\mu^{(s^2)}_{1,ss} - \mu^{(s^2)}_{2,ss} \geq 0$ and $H_0 : \mu^{(s^2)}_{1,ws} - \mu^{(s^2)}_{2,ws} \geq 0$.

Distributions	Strong SOSD	Weak SOSD
Statistic	-1.02	-0.20
p-value	0.00	0.02

Finally, we compare distributions of average second round exposure to payoff risk across the 5 observed classes of response functions. Their densities, estimated using Bernstein polynomials, in the decision tasks with payoff distributions ordered according to second-order stochastic dominance, are provided in Figure 5. Consistent with

[41]The average second round payoff variance of individuals in Classes 1 and 2 are 1.20 Euros^2 and 2.21 Euros^2, respectively.

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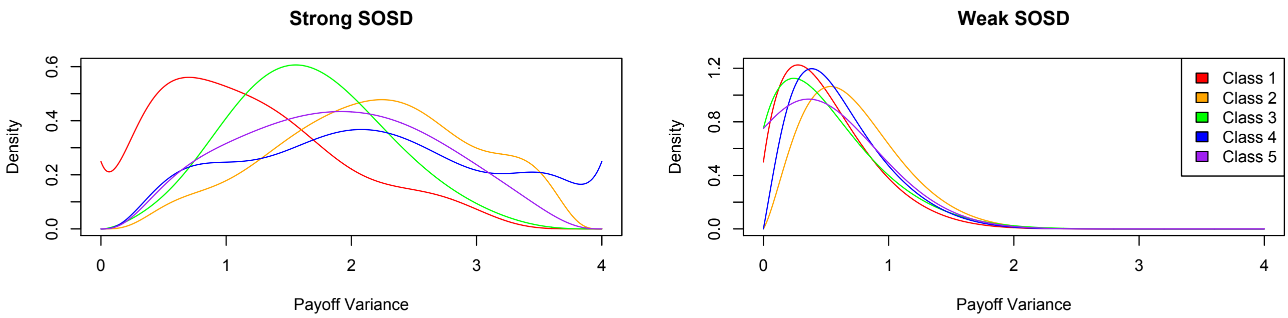

Figure 5: Second round payoff variance densities, across subjects, estimated using Bernstein polynomials.

the results above, Class 1 subjects' variance densities concentrate more probability to the left than the variance densities of Class 2 subjects. It can also be seen that Class 3 subjects' densities concentrate more probability to the left than the densities of other classes, which is consistent with them being the class, outside of Class 1, that has the biggest chance of choosing the same alternative in both rounds when the first round payoff is 3.

Overall, these results illustrate how the mean level and exposure to risk of a subject's second round payoffs depend critically on her response function.

6 Discussion

In Subsection 6.1 we discuss how the response function classes reported in Section 4 might be expected to perform in settings more general than that of our experiment. In Subsection 6.2 we explore possible determinants of individuals' response functions. In Subsection 6.3 we discuss the relationship between the shapes of the response function and Bernoulli utility function of an expected utility maximizer, as well as possible extensions to our work.

6.1 Interpreting performance

It is natural to ask how the different response function classes reported in Section 4 would have performed if we had chosen different pairs of payoff distributions in our experimental design.[42] Whereas the theory results presented in Section 2 indicate that increasing response functions of maximum strength typically perform well relative to alternatively shaped response functions, they also suggest that the relative performance of a response function is sensitive to the specific pair of unknown payoff distributions that underlie alternatives. We illustrate these points below.

In Section 5 we report that response function Classes 1 and 2 earn substantially higher payoffs than Classes 3, 4, and 5 in the decision tasks in which the payoff distributions are ordered by the first-order stochastic dominance

[42]Recall from Section 3 that this is a valid counterfactual exercise given that the elicitation of the response functions that we report in Section 4 is insensitive to the specific pairs of payoff distributions that we selected, which were unknown to the subjects in the experiment.

relationships reported in Table 1. Subjects in these classes appear to use the middle payoff of 3 Euros as a type of threshold: if the first round payoff is below 3 then they switch alternatives in the second period, whereas if it is above 3 then they stay with the same alternative. From the results in Section 2.1, we know that the increasingness of these response functions (Proposition 1), and the maximum strength (Proposition 2), both generally lead to relatively higher payoffs. However, these results do not indicate whether the threshold should be at 3, as opposed to above or below 3. In particular, it is easy to come up with pairs of payoff distributions that are ordered according to first-order stochastic dominance, such that Classes 1 and 2 are outperformed by other response functions.[43] Fortunately, it is possible to "reverse engineer" the possible pairs of distributions for which observed behavior performs the best. In particular, it is easy to see that subjects in Classes 1 and 2 perform the best for pairs of payoff distributions in which the dominant distribution places all probability mass above the threshold of 3, and the dominated distribution places all probability mass below the threshold.

The above discussion, and the results reported in Section 5, assume a stochastic dominance ordering across payoff distributions. Once we drop this assumption, it is natural to ask again how to interpret the more general performance potential of Classes 1 and 2. Here, Proposition 3 and Corollary 2 give us guidance. In particular, increasing response functions that are a positive affine transformation of an individual's underlying Bernoulli utility function lead to the preferred payoff distribution being chosen more often than the less preferred distribution, and this effect is maximized when the response function is stretched to maximum strength. With these results in mind, we observe that the response functions of Classes 1-3 seem roughly similar to the shapes we would expect of Bernoulli utility functions, with Classes 1 and 2 stretched to near maximum strength. However, the relative flatness of the response functions of Classes 1 and 2 between payoffs 1 and 2, and between payoffs 4 and 5, might favor a Bayesian interpretation of response functions, as discussed below. On the other hand, the response functions of Classes 4 and 5 are far from maximum strength, and seem relatively less similar to the types of Bernoulli utility functions that we would expect to see, with Class 5's response function decreasing.

Finally, to the extent that it is reasonable to assume that Bayesian Expected Utility reasoning underlies the individuals' response functions that we observe, the theory results from Section 2.2 suggest that response functions should be dichotomic between stay and switch, and (weakly) increasing if beliefs satisfy the MLRP.

[43] For instance, consider a modified version of our Strong FOSD pair of distributions, in which the dominant distribution, instead of placing 96% of the probability mass on $x = 5$, placed it on $x = 2$; notice that with this change, this distribution still first-order stochastically dominates the other distribution. For such decision tasks our strongly responsive subjects would "over-experiment" in the sense that a first round payoff of 2 would trigger her to switch alternatives in the second round, and thus switch out of the dominant distribution. Similarly, if we had instead shifted the probability mass of the dominated distribution from 1 to 4, our strongly responsive subjects would "under-experiment" in the sense that a first round payoff of 4 would trigger them to stay with the same alternative in the second round, though this alternative is actually dominated. If subjects were to have response functions that were increasing over $\{1, 2\}$ and increasing over $\{4, 5\}$, this would mitigate the effects of "over" and "under" experimentation, thus making these subjects relatively more likely to choose a dominant distribution in the second round. One example of such a response function is the linear response function defined by $\theta(x) = .25(x-1)$ for $x = 1, 2, ..., 5$. The average second round payoff of the linear response function in decision tasks containing Weak FOSD payoff distributions, however, would be inferior to that of the subjects in Classes 1 and 2.

This is roughly consistent with what we observe from the response functions in Classes 1 and 2, and inconsistent with Classes 3-5. Having said this, it is not impossible to come up with systems of beliefs such that response functions are non-monotone (see Example 2 in Appendix B), or even decreasing (see, e.g., Charness and Levin (2005)).

6.2 Determinants of response function class

In Section 4 we report the distribution of response function classes observed in our experiment. Here we first explore whether information collected in a post-experiment questionnaire (demographic variables), as well as responses to four financially incentivized comprehension questions about the instructions, can be used to predict a subject's response function class. In addition, we provide some psychological interpretations for the behavior of the different classes.

Table 7 reports the estimates of a logistic regression with two dependent variable categories: 0 (base)—non-to-weakly responsive subjects (Classes 3, 4, and 5), and 1—strongly responsive subjects (Classes 1 and 2), and independent variables that consist of the demographic variables Sex (1 for male and 0 for female), GPA (aggregate in-major GPA, with marks from 5 to 9), EconBusiness (1 for economics or business major, and 0 for other major), Course (course year in major, from 1 to 7), and Eduparents (the composite education level of parents; rated from 0 to 6 where none is 0, primary is 1, secondary is 2, and university is 3, for each parent), as well as dummy regressors for a correct answer (1) in each of the four comprehension test questions: Q1 tested understanding that payoffs were random variables with probability mass possibly spread across the full support, Q2 tested understanding that payoff distributions changed from one task to the next, while Q3 and Q4 tested understanding and reasoning about the stationarity of alternatives' payoff distributions across rounds of the same task. In particular, Question 4 is precisely a test of whether stationarity is being neglected as the result of Gambler's Fallacy type beliefs—that, for example, upon yielding a low (high) first round payoff, the payoff distribution then experiences a FOSD-increasing (decreasing) shift (see, e.g., Tversky and Kahneman (1971) and Rabin (2002))—and Question 3 is closely related (see Appendix E for the questions).[44]

The results displayed in Table 7 suggest that the demographic variables have no effect on determining assignation to response function class. By contrast, we find that responding correctly to question 3 or 4 increases the probability of being assigned to a strongly responsive class (both significant at the 5% level), whereas questions 1 and 2 have no effect.[45,46] These results suggest that behavioral biases such as the Gambler's Fallacy

[44] 6 of our 100 subjects had a different version of one of the questions in the comprehension test, so they are excluded from the analysis.

[45] The average marginal effects of Q3 and Q4 on the probability of being assigned to the strongly responsive classes are substantial: 0.27 and 0.33, respectively.

[46] If only the comprehension test questions are included as regressors, the estimated coefficients are similar; only questions 3 and 4 have significant effects, at the 1% and 5% levels, respectively. If only the demographic variables are included, the estimated

Table 7: Logistic regression for subjects' assignation to non-to-weakly responsive (0; base) versus strongly responsive (1) response functions.

Regressor	Coefficient	Standard error	p-value
Q1	-0.88	0.58	0.13
Q2	0.13	0.89	0.89
Q3	1.21*	0.49	0.01
Q4	1.59*	0.79	0.04
Sex	-0.48	0.50	0.33
GPA	0.16	0.33	0.62
EconBusiness	-0.41	0.55	0.46
Course	0.12	0.18	0.51
Eduparents	0.09	0.15	0.56
Constant	-2.80	2.60	0.28

Note: Statistical significance (at the 5% level) is denoted by *.

could potentially explain the presence of less responsive response functions. Further, that these misbeliefs persisted despite us (i) taking great effort in the instructions to eradicate them, and (ii) providing direct financial incentives for subjects to overcome them (in the comprehension test), suggests that, in accordance with previous research (see, e.g., Bar-Hillel and Wagenaar (1991)), these biases lie deep and are difficult to uproot. In particular, Miller and Sanjurjo (2015) show how (correct) observation of the alternation rate in finite sequences generated by repeated trials of an i.i.d. binary (or binarized) random variable can lead to the persistence of Gambler's Fallacy beliefs.[47]

A complementary explanation of the relatively high degree of non-responsiveness of Classes 4 and 5, but that does not explain the decreasing slope of Class 5 response functions (as the Gambler's Fallacy explanation can), is suggested by a very simple Bayesian interpretation. In particular, perhaps the simplest assumption on beliefs would be, following the principle of insufficient reason, that both payoff distributions are uniform. In this case, the prediction is that the second period choice can be anything, as subjects are indifferent between staying with the same alternative, or switching, regardless of the experienced payoff. This indifference might lead the subject to sometimes, or even usually, randomize (uniformly) between the alternatives.

Similarly, an individual could have simple beliefs in which only two (different) distributions are possible, and each distribution is assigned to the chosen, and unchosen, alternatives with equal probabilities. If these two distributions satisfy MLRP, then by Corollary 3 of Appendix B, the response function will be increasing and essentially dichotomic. This is one way of providing an explanation for the response functions of Classes 1 and 2.

This leaves Class 3, which is increasing, as in Classes 1 and 2, but with slightly less responsiveness to high payoffs (4 and 5) and substantially less responsiveness to low payoffs (1 and 2). Overall, this is the most likely

coefficients are similar, and none are significant.

[47]This statement is true for finite sequences of length three and higher.

class to repeat the first period choice in the second period, which it does with an average probability of 0.71 across first period payoffs.[48] A possible psychological explanation for this tendency to stay with the same alternative, rather than switch, is anticipated regret (see, e.g., Loomes and Sugden (1982)). The interpretation has been used, for instance, to provide a satisfying explanation for the robust finding of peoples' surprisingly high error rates in the Monty Hall problem.[49] This is also one way of interpreting the tendency of subjects in Class 1 to almost always stay with the same alternative after obtaining the middle payoff of $x = 3$ in the first period.

6.3 Extensions and relation with EUT

A question of interest is how the shapes of an individual's response function and Bernoulli utility function relate. In particular, it may seem intuitive that the two functions would share the same shape. Consistent with this intuition, Proposition 3 of Section 2.1 establishes that if the response function and utility function share the same shape, then the individual's second period choice is more likely to be the alternative with the payoff distribution that the individual would prefer if the distributions were known. To what extent these two functions share the same shape in practice is an open empirical question.

A second question of interest is whether the response functions that we elicit in our experiment are better thought of as belief-free "primitive" reactions to experienced payoffs, or the consequence of imagined priors and updating on these priors given the obtained first period payoff. Our data does not allow us to separate between these two alternative interpretations. As a result, in future work it would be interesting to conduct similar experiments in which the information environment is changed in ways that would affect the shape of the response function of a Bayesian in predictable ways, e.g. by providing additional information about payoff distributions, while affecting a belief-free decision-maker perhaps not at all, allowing for the data to separate the two types.

On the other hand, if we restrict attention strictly to minimal information environments, such as the one studied here, then it may be natural to question whether the distribution of response function classes that we observe in this study also describes behavior in a broader class of choice tasks. While this question can only be fully answered by future research, we see no reason to expect that it would not, given that our design is neutral, not itself encouraging response functions of any particular shape (see Section 3). To the extent that we can in the future converge on a representative distribution of classes, and understand the relationships existing

[48] Classes 1, 2, 4, and 5 have average repeat probabilities of 0.59, 0.50, 0.44, and 0.59, respectively.

[49] A robust finding in the Monty Hall problem is that when subjects choose between staying with an initially chosen door or switching, they tend to incorrectly stay with the same door roughly 80-90% of the time (see, e.g., Friedman (1998)). This behavior is puzzling because the standard incorrect reasoning would make the subject indifferent between choosing either of the two doors, which would suggest that around 50% of incorrect subjects might stay with the same door, while *all* correct subjects will switch. The fact that subjects' observed stay rate is substantially higher than what is expected can be explained by most of those subjects who are otherwise indifferent between the two choices deciding to stay, so as to minimize anticipated regret.

between the payoff distributions of real-word choice alternatives, we can then make even stronger judgements about the fitness of the response functions that we observe. For now, we consider the observed distribution of classes, viewed through the lens of our theoretical results, to be an informative first step.

References

Agastya, M. and A. Slinko (2015): "Dynamic choice in a complex world," *Journal of Economic Theory*, 158, 232–258.

Amemiya, T. (1985): *Advanced Econometrics*, Harvard University Press.

Anderson, C. M. (2012): "Ambiguity aversion in multi-armed bandit problems," *Theory and decision*, 72, 15–33.

Anscombe, F. J. and R. J. Aumann (1963): "A definition of subjective probability," *Annals of mathematical statistics*, 199–205.

Apesteguia, J. and M. Ballester (2017): "Monotone stochastic choice models: The case of risk and time preferences," forthcoming in *Journal of Political Economy.*

Babu, J., A. Canty, and Y. Chaubey (2002): "Application of Bernstein polynomials for smooth estimation of a distribution and density function," *Journal of Statistical Planning and Inference*, 105, 377–392.

Banks, J., M. Olson, and D. Porter (1997): "An experimental analysis of the bandit problem," *Economic Theory*, 10, 55–77.

Bar-Hillel, M. and W. A. Wagenaar (1991): "The perception of randomness," *Advances in applied mathematics*, 12, 428–454.

Barron, G. and I. Erev (2003): "Small feedback-based decisions and their limited correspondence to description-based decisions," *Journal of Behavioral Decision Making*, 16, 215–233.

Benartzi, S. (2001): "Excessive extrapolation and the allocation of 401 (k) accounts to company stock," *The Journal of Finance*, 56, 1747–1764.

Benjamini, Y. and Y. Hochberg (1995): "Controlling the false discovery rate: a practical and powerful approach to multiple testing," *Journal of the Royal Statistical Society. Series B (Methodological)*, 289–300.

Börgers, T., A. Morales, and R. Sarin (2004): "Expedient and monotone learning rules," *Econometrica*, 72, 383–405.

Bush, R. and F. Mosteller (1951): "A mathematical model of simple learning," *Psychological Review*, 58, 313–323.

——— (1955): *Stochastic Models for Learning.*, John Wiley & Sons, Inc.

Charness, G. and D. Levin (2005): "When optimal choices feel wrong: A laboratory study of Bayesian updating, complexity, and affect," *American Economic Review*, 1300–1309.

Choi, J., D. Laibson, B. Madrian, and A. Metrick (2009): "Reinforcement learning and savings behavior," *The Journal of Finance*, 64, 2515–2534.

DeGroot, M. H. (2005): *Optimal statistical decisions*, vol. 82, John Wiley & Sons.

Dempster, A., N. Laird, and D. Rubin (1977): "Maximum likelihood from incomplete data via the EM algorithm," *Journal of the Royal Statistical Society Series B*, 39, 1–38.

Denrell, J. (2007): "Adaptive learning and risk taking." *Psychological Review*, 114, 177.

Easley, D. and A. Rustichini (1999): "Choice without beliefs," *Econometrica*, 67, 1157–1184.

El-Gamal, M. A. and D. M. Grether (1995): "Are people Bayesian? Uncovering behavioral strategies," *Journal of the American Statistical Association*, 90, 1137–1145.

Erev, I. and E. Haruvy (2013): "Learning and the economics of small decisions," *The handbook of experimental economics*, 2.

Erev, I. and A. Roth (1998): "Predicting how people play games: Reinforcement learning in experimental games with a unique mixed strategy equilibria," *American Economic Review*, 88, 848–881.

Fay, M. and P. Shaw (2010): "Exact and asymptotic weighted logrank tests for Interval censored data: the interval R package," *Journal of Statistical Software*, 36, 1–34.

Feltovich, N. (2000): "Reinforcement-based vs. Belief-based Learning Models in Experimental Asymmetric-information Games," *Econometrica*, 68, 605–641.

Fischbacher, U. (2007): "z-Tree: Zurich toolbox for ready-made economic experiments," *Experimental Economics*, 10, 171–178.

Friedman, D. (1998): "Monty Hall's three doors: Construction and deconstruction of a choice anomaly," *American Economic Review*, 88, 933–946.

Gilboa, I. and D. Schmeidler (1989): "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 18, 141–153.

Gonzalez, R. and G. Wu (1999): "On the shape of the probability weighting function," *Cognitive Psychology*, 38, 129–166.

Good, P. (2005): *Permutation, Parametric, and Bootstrap Tests of Hypotheses*, New York: Springer.

Grether, D. M. (1980): "Bayes rule as a descriptive model: The representativeness heuristic," *The Quarterly Journal of Economics*, 95, 537–557.

Grosskopf, B., I. Erev, and E. Yechiam (2006): "Foregone with the wind: Indirect payoff information and its implications for choice," *International Journal of Game Theory*, 34, 285–302.

Grün, B. and F. Leisch (2007): "Fitting finite mixtures of generalized linear regressions in R," *Computational Statistics and Data Analysis*, 51, 5247–5252.

——— (2008): "FlexMix Version 2: Finite Mixtures with Concomitant Variables and Varying and Constrant Parameters," *Journal of Statistical Software*, 28, 1–35.

Hey, J. D. and C. Orme (1994): "Investigating generalizations of expected utility theory using experimental data," *Econometrica*, 1291–1326.

Holt, C. A. and A. M. Smith (2009): "An update on Bayesian updating," *Journal of Economic Behavior & Organization*, 69, 125–134.

Hopkins, E. (2002): "Two competing models of how people learn in games," *Econometrica*, 70, 2141–2166.

Hu, Y., Y. Kayaba, and M. Shum (2013): "Nonparametric learning rules from bandit experiments: The eyes have it!" *Games and Economic Behavior*, 81, 215–231.

Jiang, W. and M. A. Tanner (1999): "On the identifiability of mixtures-of-experts," *Neural Networks*, 12, 1253–1258.

Kaustia, M. and S. Knüpfer (2008): "Do investors overweight personal experience? Evidence from IPO subscriptions," *The Journal of Finance*, 63, 2679–2702.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005): "A smooth model of decision making under ambiguity," *Econometrica*, 73, 1849–1892.

Loomes, G. (1998): "Probabilities vs money: a test of some fundamental assumptions about rational decision making," *The Economic Journal*, 108, 477–489.

Loomes, G. and R. Sugden (1982): "Regret theory: An alternative theory of rational choice under uncertainty," *The Economic Journal*, 92, 805–824.

——— (1995): "Incorporating a stochastic element into decision theories," *European Economic Review*, 39, 641–648.

Maniadis, Z. and J. Miller (2012): "The Weight of Personal Experience: an Experimental Measurement," Tech. rep.

March, J. (1996): "Learning to be risk averse." *Psychological Review*, 103, 309.

McLachlan, G. and K. Basford (1988): *Mixture Models: Inference and Applications to Clustering*, New York: Marcel Dekker.

McLachlan, G. and T. Krishnan (2008): *The EM Algorithm and Extensions*, New York: Wiley.

McLachlan, G. and D. Peel (2000): *Finite Mixture Models*, Wiley.

Mengel, F. and J. Rivas (2012): "An axiomatization of learning rules when counterfactuals are not observed," *The BE Journal of Theoretical Economics*, 12.

Miller, J. and A. Sanjurjo (2015): "Surprised by the Gambler's and Hot Hand Fallacies? A Truth in the Law of Small Numbers," Tech. rep.

Narendra, K. S. and M. A. Thathachar (1974): "Learning automata – a survey," *IEEE Transactions on Systems, Man and Cybernetics*, 4, 323–334.

Neilson, W. (2010): "A simplified axiomatic approach to ambiguity aversion," *Journal of Risk and Uncertainty*, 41, 113–124.

Nevo, I. and I. Erev (2012): "On surprise, change, and the effect of recent outcomes," *Frontiers in Psychology*, 3.

Norman, M. F. (1968): "On the linear model with two absorbing barriers," *Journal of Mathematical Psychology*, 5, 225–241.

Oyarzun, C. and R. Sarin (2012): "Mean and variance responsive learning," *Games and Economic Behavior*, 75, 855–866.

——— (2013): "Learning and risk aversion," *Journal of Economic Theory*, 148, 196–225.

Prelec, D. (1998): "The probability weighting function," *Econometrica*, 497–527.

R Core Team (2013): *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria.

Rabin, M. (2002): "Inference by believers in the law of small numbers," *The Quarterly Journal of Economics*, 117, 775–816.

Redner, R. and H. Walker (1984): "Mixture densities, maximum likelihood and the EM algorithm," *SIAM Review*, 26, 195–239.

Rothschild, M. and J. E. Stiglitz (1970): "Increasing risk: I. A definition," *Journal of Economic Theory*, 2, 225–243.

Selten, R., A. Sadrieh, and K. Abbink (1999): "Money does not induce risk neutral behavior, but binary lotteries do even worse," *Theory and Decision*, 46, 213–252.

Simonsohn, U., N. Karlsson, G. Loewenstein, and D. Ariely (2008): "The tree of experience in the forest of information: Overweighing experienced relative to observed information," *Games and Economic Behavior*, 62, 263–286.

Titterington, D. M., A. F. M. Smith, and U. E. Makov (1985): *Statistical Analysis of Finite Mixture Distributions*, New York: Wiley.

Tversky, A. and D. Kahneman (1971): "Belief in the law of small numbers," *Psychological Bulletin*, 76, 105.

Wilcox, N. T. (2008): "Stochastic models for binary discrete choice under risk: A critical primer and econometric comparison," in *Risk aversion in experiments*, Emerald Group Publishing Limited, 197–292.

——— (2011): "Stochastically more risk averse: A contextual theory of stochastic discrete choice under risk," *Journal of Econometrics*, 162, 89–104.

Appendix A: Proofs of Section 2.1

Proof of Proposition 2:

Proof. Suppose that the increasing response function θ_0 is not maximum strength. Define the response function $\theta_1 := \theta_0 - \theta_0(\underline{x})$. From (2), we have $\mathbb{E}P_a(\theta_1) = \mathbb{E}P_a(\theta_0)$. Now define the response function $\theta := (\theta_0(\overline{x}) - \theta_0(\underline{x}))^{-1}\,\theta_1$. Since $(\theta_0(\overline{x}) - \theta_0(\underline{x}))^{-1} > 1$, from (2) we obtain $\mathbb{E}P_a(\theta) \geq \mathbb{E}P_a(\theta_1)$, which proves that (ii) implies (i). To prove the converse, consider any maximum strength response function θ and, without loss of generality, any maximum strength response function θ'. If there exists $x \in (\underline{x}, \overline{x})$ such that $\theta'(x) > \theta(x)$, then consider the distributions F_a and $F_{a'}$ that yield $\overline{x}$ and x, respectively, with probability one. From (2), $\mathbb{E}P_a(\theta) > \mathbb{E}P_a(\theta')$. On the other hand, if there exists $x \in (\underline{x}, \overline{x})$ such that $\theta'(x) < \theta(x)$, then consider the distributions F_a and $F_{a'}$ that yield x and $\underline{x}$, respectively, with probability one. From (2), $\mathbb{E}P_a(\theta) > \mathbb{E}P_a(\theta')$. ■

Proof of Proposition 3:

Proof. From (2),

$$\mathbb{E}P_a(\theta) - \mathbb{E}P_{a'}(\theta) = \int_X b + cu(x)dF_a(x) - \int_X b + cu(x)dF_{a'}(x) = c\left(\int_X u(x)dF_a(x) - \int_X u(x)dF_{a'}(x)\right) \geq 0,$$

where the inequality follows from the individual preferring F_a to $F_{a'}$. ■

Appendix B: Examples of Bayesian Expected Utility Theory

We start with a simple belief structure in Example 1.

Example 1 *Consider two absolutely continuous distributions, G_1 and G_2, and assume that the individual believes that with probability $\frac{1}{2}$ the chosen and unchosen alternatives have distributions G_1 and G_2, respectively; otherwise they have distributions G_2 and G_1, respectively. Once the individual has chosen an alternative and obtained a payoff $x \in X$, she can use Bayes' rule to compute her updated beliefs that the payoff distributions of the chosen and unchosen alternatives are G_1 and G_2, respectively:*

$$\mu_x(G_1, G_2) := \frac{\frac{1}{2}g_1(x)}{\frac{1}{2}g_1(x) + \frac{1}{2}g_2(x)} = \frac{g_1(x)}{g_1(x) + g_2(x)} \equiv 1 - \mu_x(G_2, G_1),$$

where $g_1(x)$ and $g_2(x)$ are the densities of G_1 and G_2, respectively, at $x \in X$. We assume that either $g_1(x) > 0$, $g_2(x) > 0$, or both, for all $x \in X$.

The individual is assumed to be an expected utility maximizer with a strictly increasing Bernoulli utility function $u : X \to \mathbb{R}$. Thus, the individual prefers to choose the same alternative in the second period as in the *first if the (conditional) expected utility from staying,* $\mu_x(G_1,G_2)\int_X u(z)dG_1(z)+(1-\mu_x(G_1,G_2))\int_X u(z)dG_2(z)$, *is greater than the expected utility of switching,* $\mu_x(G_1,G_2)\int_X u(z)dG_2(z)+(1-\mu_x(G_1,G_2))\int_X u(z)dG_1(z)$, *or, equivalently, if*

$$\frac{g_1(x)-g_2(x)}{g_1(x)+g_2(x)}\left(\int_X u(z)dG_1(z)-\int_X u(z)dG_2(z)\right)\geq 0. \tag{6}$$

Thus, the individual prefers staying to switching if (i) $g_1(x)\geq g_2(x)$ *and* $\int_X u(z)dG_1(z)\geq\int_X u(z)dG_2(z)$, *or (ii)* $g_1(x)\leq g_2(x)$ *and* $\int_X u(z)dG_1(z)\leq\int_X u(z)dG_2(z)$. *In both (i) and (ii) the individual prefers switching to staying if one of the inequalities is reversed. Similarly, in both (i) and (ii) preferences for switching, or staying, become strict if both inequalities are strict.*

We now consider simple (thus arguably plausible) belief structures that an individual might hold. They ensure that response functions are not only dichotomic, but also (weakly) monotonically increasing. First, consider the case (encompassing Example 1) in which the individual believes that only two distributions are possible—G_1 and G_2—and that each distribution is assigned to the chosen, and unchosen, alternatives with equal probabilities, i.e. $\mu\left(\{(G_a,G_{a^-})=(G_1,G_2)\}\right)=\mu\left(\{(G_a,G_{a^-})=(G_2,G_1)\}\right)=\frac{1}{2}$. Suppose that the corresponding densities exist, and are denoted by g_1 and g_2, respectively. Refer to such beliefs as *binary-allocative.* Second, consider beliefs that are similar to those just described, but that instead satisfy: $\mu\left(\{(G_a,G_{a^-})=(G_1,G_2)\}\right)=\mu\left(\{(G_a,G_{a^-})=(G_2,G_1)\}\right)=\mu\left(\{(G_a,G_{a^-})=(G_1,G_1)\}\right)=\mu\left(\{(G_a,G_{a^-})=(G_2,G_2)\}\right)=\frac{1}{4}$; that is, let the assignations of payoff distributions to each alternative be independent of one another. Refer to such beliefs as *binary-independent.* The following result is a direct consequence of Observation 2:

Corollary 3 *Suppose that an individual's beliefs* μ *are binary-allocative or binary-independent. If* g_1 *and* g_2 *cross only once, at* $x=s$, *e.g., if they satisfy the monotone likelihood ratio property,*[50] *then the response function of the individual is:*

$$\theta_B(x)\begin{cases}=0 & \text{if } x<s\\ \in[0,1] & \text{if } x=s\\ =1 & \text{if } x>s.\end{cases}$$

Next, we provide an example of beliefs that result in a non-monotone response function for an expected payoff maximizer.

Example 2 *Consider an expected payoff maximizer with Bernoulli utility function* $u(x)=x$ *for all* $x\in X$. *Assume* $X=\{1,2,3,4,5\}$.[51] *Suppose that the individual believes there are two possible probability distributions*

[50] I.e., $g_1(x)/g_2(x)$ is strictly increasing on X.

[51] While throughout Section 2 we assume X to be an interval, our analysis extends in a natural way to the case in which the set of possible payoffs is a finite set of real numbers, $X:=\{\underline{x},...,\overline{x}\}$.

over payoffs, which are independent, and each occur with probability one half: $\left(0, 0, 1-p, \frac{1}{2}p, \frac{1}{2}p\right)$ *, with expected payoff* $3 + \frac{3}{2}p$*, and* $\left(\frac{1}{2}(1-p-\varepsilon), \frac{1}{2}(1-p-\varepsilon), 0, \frac{1}{2}(p+\varepsilon), \frac{1}{2}(p+\varepsilon)\right)$ *, with expected payoff* $\frac{3}{2} + 3(p+\varepsilon)$*, where* $p \in (0.5, 1)$ *and* $\varepsilon \in (0, 1-p)$*. The difference in the expected value is* $\frac{3}{2}(1-p) - 3\varepsilon$*. Thus, for small enough* ε*, the first distribution has strictly greater expected utility than the second one. Routine computations reveal that upon obtaining a payoff* $x \in \{1, 2, 4, 5\}$*, the updated beliefs that the payoff distribution of the chosen alternative is the second one are strictly greater than one half. In contrast, upon obtaining a payoff* $x = 3$*, the updated belief that the payoff distribution of the chosen alternative is the second one is* 0*. Hence* $\theta_B(x) = 0$ *if* $x \in \{1, 2, 4, 5\}$ *and* $\theta_B(3) = 1$*.*

Appendix C: Random Decision Making

We provide an example for the random parameter model presented in Section 2.3. Then we consider a random utility model.

Random Parameter Model. In the following example, which closely follows Holt and Smith (2009), the decision-maker errs when performing Bayesian updating.

Example 3 *Consider an expected utility maximizer, such as the one described in Section 2.2, who has a strictly increasing Bernoulli utility function* $u : X \to \mathbb{R}$*, and the prior beliefs described in Example 1. In addition, let* $\int_X u(z)dG_1(z) > \int_X u(z)dG_2(z)$*.*

As before, let $\mu_x(G_1, G_2)$ *be the updated probability, using Bayes' rule, that the payoff distributions of the chosen and unchosen alternatives are* G_1 *and* G_2*, respectively, once the individual has chosen an alternative and experienced a payoff of* $x \in X$*. Following Holt and Smith (2009), here we assume that the individual may misperceive the updated probability of* (G_1, G_2) *upon experiencing* x*, and we model this misperception using a weighted probability model.*[52] *In particular we assume that, instead of considering the updated probability* $\mu_x(G_1, G_2)$*, the individual considers the weighted updated probability*

$$w\left(\mu_x(G_1, G_2)\right) = \frac{e^{\delta}\mu_x(G_1, G_2)^{\gamma}}{e^{\delta}\mu_x(G_1, G_2)^{\gamma} + (1 - \mu_x(G_1, G_2))^{\gamma}},$$

where: (i) $\gamma \in \mathbb{R}_{\geq 0}$ *determines how the weighted probability* w *responds to changes in the correctly computed updated probability according to Bayes rule, and (ii)* $\delta \in \Theta = \mathbb{R}$ *may be interpreted as a bias-to-stay (with the same alternative) measure.*[53]

[52] For other references on Bayesian updating mistakes, see, e.g., Grether (1980) and El-Gamal and Grether (1995). While the use of weighted probability functions to model Bayesian updating mistakes was pioneered by Holt and Smith (2009), these functions have previously been studied in psychology and economics (see, e.g., Prelec (1998) and Gonzalez and Wu (1999)).

[53] With $\delta = 0$ and $\gamma = 1$ we obtain the Bayesian expected utility maximizer of Section 2.2 as a special case. For $\gamma > (<)1$, the

Similar to Holt and Smith (2009), here we consider γ as fixed for an individual and allow δ to vary randomly across decision tasks according to the distribution function F_δ with density f_δ.[54] *The randomness in δ yields a model of stochastic choice. Following Apesteguia and Ballester (2017) now, δ is a* random parameter*, and staying and switching are Θ-ordered.*

The individual prefers to stay if the conditional expected utility of staying is greater than that of switching, i.e., if $w(\mu_x(G_1, G_2)) > 1/2$. Therefore, $\delta^*(x) = \gamma \ln\left(\frac{1-\mu_x(G_1,G_2)}{\mu_x(G_1,G_2)}\right)$ is the cut-off value for δ such that the individual stays (switches) if δ is greater (smaller) than the cut-off, and the individual stays with probability

$$\theta_P(x) = 1 - F_\delta\left(\gamma \ln\left(\frac{1-\mu_x(G_1,G_2)}{\mu_x(G_1,G_2)}\right)\right).$$

Direct computations yield:

$$\frac{d\theta_P(x)}{dx} = f_\delta\left(\gamma \ln\left(\frac{1-\mu_x(G_1,G_2)}{\mu_x(G_1,G_2)}\right)\right)\gamma\frac{\mu'_x(G_1,G_2)}{\mu_x(G_1,G_2)\left(1-\mu_x(G_1,G_2)\right)},$$

where $\mu'_x(G_1, G_2)$ is the derivative of $\mu_x(G_1, G_2)$ with respect to x. Thus, as mentioned in Section 2.3, the slope of the response function is shaped by the density of the random parameter, f_δ, and by the rate of change of the cut-off of δ with respect to x (here corresponding to the expression $-\gamma\frac{\mu'_x(G_1,G_2)}{\mu_x(G_1,G_2)(1-\mu_x(G_1,G_2))}$). In particular, for ranges of x such that the cut-off $\delta^*(x)$ is contained in an interval where f_δ is nearly constant, the magnitude of the slope is increasing in γ. For instance, if $\mu'_x(G_1, G_2) > 0$, for greater values of γ, an increase in x causes a greater decrease in the cut-off, and hence a greater increase in the probability of staying.

Random Utility Model. In supplementing the Bayesian model from Section 2.2 with a stochastic component, one can also adopt a simple approach that has been used by experimentalists when studying decision theories under full information of payoff distributions (see, e.g., Hey and Orme (1994)). In particular, we assume that when the decision-maker chooses between staying and switching, upon experiencing $x \in X$, she will choose to stay (switch) if $U_{\text{stay}}(x) - U_{\text{switch}}(x) > (<)\,\varepsilon$, where ε is a random variable with distribution F_ε that represents errors in decision-making. Whenever we assume that F_ε is differentiable, the density of ε will be denoted by f_ε.[55]

The response function induced by this model is:

$$\theta_R(x) = F_\varepsilon(\rho(x)) = F_\varepsilon\left(U_{\text{stay}}(x) - U_{\text{switch}}(x)\right)$$

individual underweights low (high) probabilities and overweights high (low) probabilities. For a thorough analysis of this weighting function, see Gonzalez and Wu (1999).

[54] In Holt and Smith (2009), the idiosyncratic randomness is captured by an error term in their regressions.

[55] In the empirical analysis of Hey and Orme (1994) the authors find it convenient to further assume that $\varepsilon \sim N(0, 1)$ when performing empirical estimations.

for all $x \in X$. Since F_ε is weakly increasing by definition, a weakly increasing conditional expected difference $\rho(x) = U_{\text{stay}}(x) - U_{\text{switch}}(x) = \int_E U(a) - U(a^-) d\mu_x$ yields a weakly increasing response function θ_R. To provide a simple illustration, we now consider a version of Example 1 in which we allow for decision-making errors:

Example 4 *(Example 1, continued) When allowing for stochastic choice, we obtain from (6) that the individual prefers to choose the same alternative in the second period as in the first, upon experiencing $x \in X$, with probability*

$$\theta_R(x) = F_\varepsilon \left(\frac{g_1(x) - g_2(x)}{g_1(x) + g_2(x)} \left(\int_X u(z) dG_1(z) - \int_X u(z) dG_2(z) \right) \right) \tag{7}$$

In particular, if g_1 and g_2 satisfy MLRP, then $\theta_R(x)$ is non-decreasing.

We now examine how the shape of an individual's response function is affected by her beliefs. In particular, our next result shows that the slope of the response function is proportional to the conditional covariance between the percentage rate of change in the chosen alternative's payoff density at the payoff x, and the difference in the expected utility between alternatives.[56] If G_a is twice differentiable for all $(G_a, G_{a'}) \in E$, then we define

$$\mathbb{COV}_{\mu_x} \left(\frac{g_a'(x)}{g_a(x)}, U(a) - U(a^-) \right) := \int_E \left(\frac{g_a'(x)}{g_a(x)} - \int_E \frac{g_{\tilde{a}}'(x)}{g_{\tilde{a}}(x)} d\mu_x \right) (U(a) - U(a^-)) d\mu_x.$$

for all $x \in X$. The result on the slope of the response function then follows:

Proposition 4 *Suppose that F_ε is differentiable and that G_a is twice differentiable for every $(G_a, G_{a'})$ in E. Then,*

$$\frac{d\theta_R(x)}{dx} = f_\varepsilon(\rho(x)) \mathbb{COV}_{\mu_x} \left(\frac{g_a'(x)}{g_a(x)}, U(a) - U(a^-) \right) \tag{8}$$

for all $x \in X$.

Proof. It suffices to show that $\rho'(x) = \mathbb{COV}_{\mu_x} \left(\frac{g_a'(x)}{g_a(x)}, U(a) - U(a^-) \right)$, which follows from[57]

$$\begin{aligned}
\rho'(x) &= \int_E \frac{g_a'(x) \int_E g_a(x) d\mu - g_a(x) \int_E g_a'(x) d\mu}{\left(\int_E g_{\tilde{a}}(x) d\mu \right)^2} \left(U(a) - U(a^-) \right) d\mu \\
&= \int_E \frac{g_a(x)}{\int_E g_{\tilde{a}}(x) d\mu} \left(\frac{g_a'(x)}{g_a(x)} - \frac{\int_E g_{\tilde{a}}'(x) d\mu}{\int_E g_{\tilde{a}}(x) d\mu} \right) \left(U(a) - U(a^-) \right) d\mu \\
&= \int_E \left(\frac{g_a'(x)}{g_a(x)} - \int_E \frac{g_{\tilde{a}}'(x)}{g_{\tilde{a}}(x)} d\mu_x \right) (U(a) - U(a^-)) d\mu_x,
\end{aligned}$$

[56] The conditional covariance is taken across all of the possible realizations of pairs of payoff distributions, according to the individual's beliefs.

[57] For some pairs of payoff distributions $(G_a, G_{a^-}) \in E$, $g_a(x)$ may be equal to zero, making the right-hand side of the second equality undefined. However, such distributions can be removed from E in order to define a set $E_x \subset E$, in which the second equality is well-defined. In an abuse of notation, we leave the integration set as E, taking as given that those pairs of payoff distributions are removed whenever the integral is defined.

for all $x \in X$. ■

For an illustration of simple beliefs that yield a response function with positive slope, see Example 5 below. In that example, the individual considers only two possible payoff distributions to be assigned to the alternatives, and such that the derivative of one of the corresponding densities is positive, while the other is negative. This makes the expected utility of the former distribution higher than that of the latter. As a result, in the event that the chosen alternative has the payoff distribution with increasing (decreasing) density, the difference between the expected utility of the chosen and unchosen distributions, i.e. $U(a) - U(a^-)$, is positive (negative). Thus, $\mathbb{COV}_{\mu_x}\left(\frac{g_a'(x)}{g_a(x)}, U(a) - U(a^-)\right) > 0$ for all $x \in X$, so the resulting response function is increasing everywhere. By contrast, in Example 6 we show that one can come up with beliefs that result in a response function that is non-monotone. In this case, the density of the payoff distribution that has the highest (lowest) expected utility is decreasing (increasing) across some payoff intervals, hence yielding $\mathbb{COV}_{\mu_x}\left(\frac{g_a'(x)}{g_a(x)}, U(a) - U(a^-)\right) < 0$ and that the response function is decreasing over the corresponding parts of the domain.

Example 5 *(Example 4, continued) Assume that the individual believes there are only two possible distributions, whose densities are* $g_1(x) = \frac{1}{2}x$ *and* $g_2(x) = 1 - \frac{1}{2}x$*, respectively, for all* $x \in [0, 2]$*. Further, assume that* $\mu(\{g_a = g_1 \text{ and } g_{a^-} = g_2\}) = \mu(\{g_a = g_2 \text{ and } g_{a^-} = g_1\}) = \frac{1}{2}$*. Let* $U_1 := \int_X u(z)dG_1(z)$ *and* $U_2 := \int_X u(z)dG_2(z)$*. Because* $\frac{g_1}{g_2}$ *is strictly increasing,* G_1 *first-order stochastically dominates* G_2 *and the expected utility of the payoff distribution associated with the density* g_1 *is strictly larger than the expected utility associated with* g_2*. It follows that, in the event* $\{g_a = g_1 \text{ and } g_{a^-} = g_2\}$, $\frac{g_a'(x)}{g_a(x)} > 0$ *and* $U(a) > U(a^-)$*, whereas in the complimentary event* $\{g_a = g_2 \text{ and } g_{a^-} = g_1\}$, $\frac{g_a'(x)}{g_a(x)} < 0$ *and* $U(a) < U(a^-)$*. Thus,* $\mathbb{COV}_{\mu_x}\left(\frac{g_a'(x)}{g_a(x)}, U(a) - U(a^-)\right) > 0$*. From (7),* $\theta_R(x) = F_\varepsilon((x-1)(U_1 - U_2))$ *for all* $x \in [0, 2]$*. Finally, if we assume that* ε *is uniformly distributed over* $[U_2 - U_1, U_1 - U_2]$*, then*

$$\theta_R(x) = \frac{1}{2(U_1 - U_2)}((x-1)(U_1 - U_2) - (U_2 - U_1)) = \frac{1}{2}x$$

for all $x \in [0, 2]$*.*

Example 6 *Assume that the individual believes there are only two possible distributions, whose densities are:*

$$g_3(x) = \begin{cases} \frac{1}{2} - x & \text{if } 0 \le x \le \frac{1}{2} \\ x - \frac{1}{2} & \text{if } \frac{1}{2} < x \le \frac{3}{2} \\ \frac{5}{2} - x & \text{if } \frac{3}{2} < x \le 2. \end{cases}$$

and $g_4(x) = 1 - g_3(x)$*, respectively, for all* $x \in [0, 2]$*. Further, assume that* $\mu(\{g_a = g_3 \text{ and } g_{a^-} = g_4\}) = \mu(\{g_a = g_4 \text{ and } g_{a^-} = g_3\}) = \frac{1}{2}$*. Let* $U_3 := \int_X u(z)dG_3(z)$ *and* $U_4 := \int_X u(z)dG_4(z)$*. Standard computations* *reveal that* G_3 *first-order stochastically dominates* G_4*. For all* $x \in \left(\frac{1}{2}, \frac{3}{2}\right)$*, in the event* $\{g_a = g_3 \text{ and } g_{a^-} = g_4\}$*, we have* $\frac{g'_a(x)}{g_a(x)} > 0$ *and* $U(a) > U(a^-)$*, whereas in the complimentary event* $\{g_a = g_4 \text{ and } g_{a^-} = g_3\}$*, we have* $\frac{g'_a(x)}{g_a(x)} < 0$ *and* $U(a) < U(a^-)$*; thus,* $\mathbb{COV}_{\mu_x}\left(\frac{g'_a(x)}{g_a(x)}, U(a) - U(a^-)\right) > 0$*. On the other hand, for all* $x \in \left(0, \frac{1}{2}\right) \cup \left(\frac{3}{2}, 2\right)$*, in the event* $\{g_a = g_3 \text{ and } g_{a^-} = g_4\}$*, we have* $\frac{g'_a(x)}{g_a(x)} < 0$ *and* $U(a) > U(a^-)$*, whereas in the complimentary event* $\{g_a = g_4 \text{ and } g_{a^-} = g_3\}$*, we have* $\frac{g'_a(x)}{g_a(x)} > 0$ *and* $U(a) < U(a^-)$*; thus,* $\mathbb{COV}_{\mu_x}\left(\frac{g'_a(x)}{g_a(x)}, U(a) - U(a^-)\right) < 0$*. From (7),* $\theta_R(x) = F_\varepsilon\left(\left(2g_3(x) - 1\right)\left(U_3 - U_4\right)\right)$ *for all* $x \in [0, 2]$*. Finally, if we assume that* ε *is uniformly distributed over* $[U_4 - U_3, U_3 - U_4]$*, then*

$$\theta_R(x) = \frac{1}{2\left(U_3 - U_4\right)}\left(\left(2g_3(x) - 1\right)\left(U_3 - U_4\right) - \left(U_4 - U_3\right)\right) = g_3(x)$$

for all $x \in [0, 2]$*.*

Figure 6 shows the densities of the distributions considered by the beliefs in Examples 5 and 6. In the graph on the left panel, the payoff distribution that yields the highest expected utility has an increasing density. Thus, these beliefs result in an increasing response function. On the other hand, in the graph on the right panel, the payoff distribution that yields the highest expected utility has a decreasing density for extreme low, and high, payoffs x. Thus, the covariance between $g'_a(x)/g_a(x)$ and $U(a) - U(a^-)$ is negative. As a consequence, for extreme low or high payoffs, the derivative of the response function with respect to the first-period payoff x is negative as well.

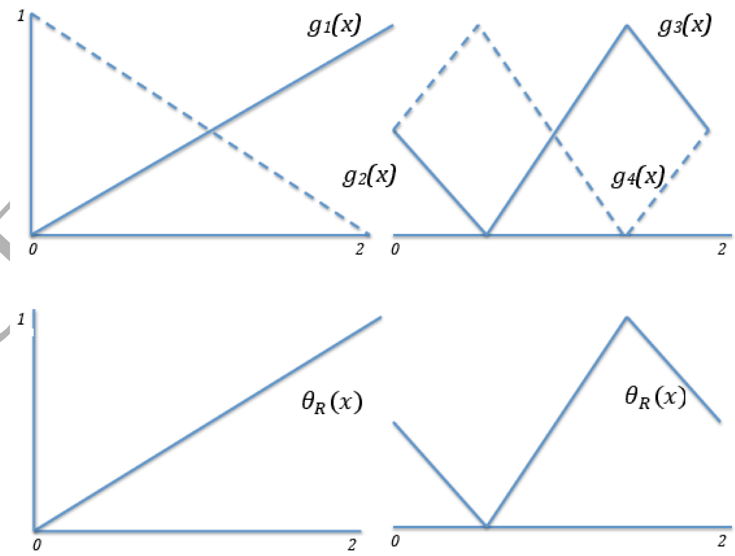

Figure 6: Densities of beliefs and response functions in Examples 5 (left panel) and 6 (right panel).

Finally, we briefly discuss the two factors that determine the slope of the response function, which are given on the right hand side of Equation (8). We start with the density of the decision-making error f_ε. Consider two individuals, j and k, who have identical preferences u and beliefs μ, and decision-making error distributions (densities) given by F_ε^j and F_ε^k (f_ε^j and f_ε^k), respectively. It follows from (8) that if $f_\varepsilon^k(\epsilon) < f_\varepsilon^j(\epsilon)$ for all $\epsilon \in (\underline{\rho}, \overline{\rho})$, where $\underline{\rho} = \inf_{x \in X} \rho(x)$ and $\overline{\rho} = \sup_{x \in X} \rho(x)$, then $\left|\frac{d\theta_R^k(x)}{dx}\right| \leq \left|\frac{d\theta_R^j(x)}{dx}\right|$ for all $x \in X$.[58] The second factor on the right hand side of Equation (8) is the (conditional) covariance term. The magnitude of the covariance is determined by the payoff distributions over which the decision-maker holds beliefs. If the payoff distributions are all similar (dissimilar) to one another, then there will be little (great) variation in the rates of change of the densities $g_a'(x)/g_a(x)$, and the difference in expected utilities $U(a) - U(a^-)$. The less (more) variation there is, the smaller (larger) the impact of changes in the obtained payoff $x \in X$ will be on $\rho(x)$, and the flatter (steeper) the response function will be.

Appendix D: Ambiguity aversion

In the general belief-based setup provided in Section 2.2, individuals: (i) have prior beliefs μ regarding the possible pairs of payoff distributions that may underlie the two alternatives that they choose between, and (ii) upon experiencing a payoff of $x \in X$, beliefs are updated to μ_x according to Bayes' rule. We now extend the analyses presented in Sections 2.2 and 2.3, and Appendixes B and C, to allow for the possibility that the decision-maker has ambiguity preferences. In particular, if the individual is ambiguity averse, then her preferences are sensitive to the uncertainty with respect to the underlying distributions of each alternative, which may affect the shape of her response function.

The approach that we adopt for incorporating ambiguity preferences is second-order expected utility (see, e.g., Neilson (2010)).[59] The preferences of the decision maker are represented by the functions $u : X \to \mathbb{R}$ and $w : u(X) \to \mathbb{R}$, such that the individual prefers staying to switching if and only if

$$W_{\text{stay}}(x) := \int_E w\left(\int u(z)dG_a(z)\right) d\mu_x \geq \int_E w\left(\int u(z)dG_{a^-}(z)\right) d\mu_x =: W_{\text{switch}}(x),$$

where concavity in u indicates risk aversion and concavity in w indicates ambiguity aversion. It then follows immediately that a result analogous to Observation 2 holds in this setting.

Next, we provide an example of the effects that ambiguity aversion can have on an individual's response function. In the example, if one payoff is highly informative about the chosen alternative's underlying payoff distribution, then such an alternative can be favored more by an ambiguity averse than an ambiguity neutral

[58] For instance if ρ is strictly increasing, for any two payoffs $x < x'$, the difference between the increases of the likelihood of staying from experiencing the higher payoff x' instead of x for individuals j and k is $\int_x^{x'} f^j(z) - f^k(z)dz > 0$, yielding that the response function of j is more responsive than the response function of k to a payoff increase.

[59] Second-order expected utility is a specific approach to model ambiguity aversion. Other approaches yield results that are qualitatively similar to those discussed here.

Expected Utility maximizer. In particular, upon experiencing such a payoff the former decides to stay with the same alternative, while the latter may switch.

Example 7 *Let an individual's preferences be given by* $w(z) = z^{1/\alpha}$ *and* $u(x) = x^{1/\rho}$*, for* $z, x \geq 0$*, where* $\alpha > 0$ *and* $\rho > 0$ *indicate ambiguity aversion and risk aversion, respectively. For each of the alternatives, the individual's prior beliefs place probabilities* 0.44*,* 0.28*, and* 0.28 *on the payoff distributions* $(0, 0, 0, 0.5, 0.5)$*,* $(0, 0, 1, 0, 0)$*, and* $(0.5, 0.5, 0, 0, 0)$*, respectively, where the entries in each payoff distribution correspond to the probabilities of obtaining a payoff of* $1, 2, 3, 4,$ *and* 5*, respectively. Further, assume that the distributions of each alternative are independent of one another.*

Now, consider a risk-averse individual, with $\rho = 2$*, who is ambiguity neutral, i.e.* $\alpha = 1$*. Standard computations reveal that* $W_{stay}(x) < W_{switch}(x)$ *for* $x = 1, 2, 3$*, whereas* $W_{stay}(x) > W_{switch}(x)$ *for* $x = 4, 5$*. Intuitively, low first period payoffs of* 1 *and* 2 *reveal that the chosen alternative's distribution is the worst, whereas high first period payoffs of* 4 *and* 5 *reveal that it is the best. On the other hand, a payoff of* $x = 3$ *reveals that the chosen alternative has the middle distribution. Finally, because the prior for each of the alternatives deems the best distribution much more likely than the middle and worst distributions, upon obtaining a first period payoff of* $x = 3$ *the individual prefers switching alternatives to staying.*

Next, if we instead assume that the individual is ambiguity averse, with, for instance, $\alpha = 4$*, standard computations reveal that whereas the preferences between staying and switching remain the same as in the case of ambiguity neutrality for first period payoffs of* $x = 1, 2, 4, 5$*, when* $x = 3$ *now* $W_{stay}(x) > W_{switch}(x)$*. Thus, ambiguity aversion leads the individual to forego the higher expected payoffs that would be obtained by switching alternatives in the second period, in favor of being certain that the payoff distribution she faces is the middle one.*

One can also use a random parameter approach to introduce randomness in the presence of ambiguity aversion. Here we allow a parameter of the model describing the second-order expected utility preferences above to vary randomly. For instance, one could consider the functions $u : X \to \mathbb{R}$ and $w : u(X) \times \Theta \to \mathbb{R}$, such that the individual prefers staying to switching if and only if $W_{\text{stay}}(x, \omega) \geq W_{\text{switch}}(x, \omega)$, i.e.,

$$\int_E w\left(\int u(z) dG_a(z), \omega\right) d\mu_x \geq \int_E w\left(\int u(z) dG_{a^-}(z), \omega\right) d\mu_x, .$$

for all $(x, \omega) \in X \times \Theta$. If we make the assumptions analogous to (i), (ii), and (iii) in the analysis of the random parameter model in Section 2.3, then the response function we obtain is given by the same expression as in (3). And, if we further assume that $W_{\text{stay}}(x, \omega)$ and $W_{\text{switch}}(x, \omega)$ are continuously differentiable over $X \times \Theta$, and $W_{\text{stay}}(x, \omega) - W_{\text{switch}}(x, \omega)$ have a strictly positive partial derivative with respect to ω, then the slope of the response function is given by an expression analogous to (4). Our next example extends Example 7 to allow for random parameters.

Example 8 *Consider the same setup described in Example 7, but now instead of setting a fixed value of α (the parameter that determines the degree of ambiguity aversion), we assume that this parameter varies randomly, taking values in the set $\Theta = (0, \infty)$ and has a distribution F_α. The same computations from Example 7, yield $W_{stay}(x, \omega) > W_{switch}(x, \omega)$ for $x = 4, 5$ and $W_{stay}(x, \omega) < W_{switch}(x, \omega)$ for $x = 1, 2$, for all $\omega \in \Theta$. For $x = 3$ the individual prefers to stay (switch) if $\alpha > (<)\alpha^* \approx 2.11$. Thus, the cut-off values of α are $\alpha^*(1) = \alpha^*(2) = \infty$, $\alpha^*(4) = \alpha^*(5) = 0$, and $\alpha^*(3) = 2.11$. Thus, staying and switching are Θ-ordered, and $\theta_P(1) = \theta_P(2) = 0$, $\theta_P(4) = \theta_P(5) = 1$, and $\theta_P(3) = 1 - F_\alpha(\alpha^*)$.*

It is also straightforward to extend the random utility model to allow for ambiguity preferences. In particular, after experiencing the first period payoff $x \in X$, the individual will choose to stay (switch) if $W_{\text{stay}}(x) - W_{\text{switch}}(x) \geq (<)\varepsilon$, where ε is a random variable that satisfies the assumptions laid out in Appendix C. Thus, the induced response function is $\theta_{RA}(x) := F_\varepsilon\left(W_{\text{stay}}(x) - W_{\text{switch}}(x)\right)$ for all $x \in X$. Then, as in the analysis of Appendix C, θ_{RA} is non-decreasing if and only if $\rho_W(x) := W_{\text{stay}}(x) - W_{\text{switch}}(x) = \int_E w\left(\int u(z)dG_a(z)\right) - w\left(\int u(z)dG_{a^-}(z)\right) d\mu_x$ is non decreasing in x. In particular, under the assumptions of Proposition 4,

$$\frac{d\theta_{RA}(x)}{dx} = f_\varepsilon(\rho_W(x))\mathbb{COV}_{\mu_x}\left(\frac{g_a'(x)}{g_a(x)}, w\left(U(a)\right) - w\left(U(a^-)\right)\right)$$

for all $x \in X$, such that $\int_E g_a(x)d\mu > 0$. This yields a result analogous to Proposition 4 but now allowing for ambiguity preferences.

Appendix E: Instructions

Welcome![60]

Before we get started, please make sure that your cell phones are turned off and that your desktops are empty.

Overview

In this experiment you will make choices in 40 **tasks**. You will be paid according to what choices you make. The experiment should take around 45 minutes. After finishing the instructions you will make choices in 3 non-paid practice tasks. After the practice tasks you will then take a short **quiz**, consisting of four questions which test your understanding of the instructions of the experiment. For each correct answer in the quiz you will receive 25 cents, in addition to your payoffs in the paid experiment that follows.

[60] The original instructions are in Spanish.

Once you finish the quiz, you will have an opportunity to ask questions before beginning the experiment. At this point, if you have a question, please raise your hand and remain silent. I will then come to you and answer your question. Once there are no more questions we will begin the experiment in which you will make choices in 40 tasks.

Upon finishing the 40 paid tasks you will be asked to fill out a short questionnaire. Once you have finished the questionnaire we will calculate your payoffs. Finally, once your payoffs are calculated we will pay you, and you are free to leave.

While we read through the instructions please keep in mind that there is a strict culture of honesty in economics experiments, so throughout this experiment you will not be tricked in any way.

It is also very important that you do not look at other students' monitors and do not communicate with any other student until the experiment is over, and you have left the computer room. If you violate either of these rules, unfortunately you will be asked to leave and you will not receive any money.
In the remainder of the instructions I will first tell you more about the choices you will be making, then I will describe how your payoffs are determined.

Choice tasks

In each of the 40 choice tasks, there are two rounds, **Round 1** and **Round 2**. In each round you will make one choice between two alternatives. Each alternative will be represented on your computer screen by a rectangle.

Round 1: In Round 1 you will use the mouse to click on the alternative of your choice. In this alternative there will then appear a number which will be your Round 1 payoff. This payoff will then disappear and you will see the same two alternatives that you saw at the beginning of the round.

Round 2: In Round 2 you will again use the mouse to click on the alternative of your choice. In Round 2, you will also receive a payoff from the chosen alternative, but unlike in Round 1, your Round 2 payoff will not appear in the alternative (we will show you this payoff at the end of the experiment if you wish to see it). After you make your Round 2 choice you will then move on to the next choice task (once all other participants have also chosen an alternative in Round 2).

Here is what you know about the two alternatives you are choosing between in each choice task:

1) The possible payoffs are 1,2,3,4, or 5 Euros.

2) For each alternative, each possible payoff has a (positive) probability.

3) For each alternative the probabilities of the 5 possible payoffs must sum to 1 (that is to say, each alternative has a "probability distribution").

4) The probability distribution is always different from one alternative to the other.

5) Each alternative has the same probability distribution in Round 1 and Round 2.

6) The probability distribution of an alternative can generate a payoff in Round 2 that is either the same as, or different than, its Round 1 payoff.

The 6 conditions that we have just reviewed are true of every choice task; recall that each task consists of two rounds. After finishing each task you will proceed to the next task. From one ***task*** to the next, the probability distribution of each alternative will change. That is, within the same choice task, for each of the alternatives, the payoff distribution in Round 1 is the same as the payoff distribution in Round 2. Then, when moving from one task to the next, the payoff distributions for both alternatives change. Once you reach the next task, the above 6 conditions hold again. In a moment, we will review these conditions once more.

Then, when moving from this task to the next, the payoff distributions for both alternatives will change again, etc.

Keep in mind that in each choice task we have chosen the relative position in which the alternatives appear on your screen at random, which means that nothing that you have observed in previous choice tasks is useful for choosing between the two alternatives in Round 1. Nevertheless, the payoff that you observe in Round 1, regardless of which alternative you chose, gives you information for your Round 2 choice in that same choice task: although you do not know the probability distributions of each alternative when you make your Round 1 choice, the payoff that you observe shows you something about the alternative you selected. For example, imagine that you have chosen the alternative on the left in Round 1, and that your payoff is 3 Euros. The information that you have just received is limited, because now you have just learned a little bit about the probability distribution of the alternative on the left, and you have learned nothing more about the probability distribution of the alternative on the right. Nevertheless, you can use this information in your Round 2 choice of alternative: if you want the distribution of payoffs in Round 2 to be the same as the distribution that generated the observed Round 1 payoff, then you can choose the same alternative. If you want the distribution of payoffs in Round 2 to be different than the distribution that generated the observed Round 1 payoff, then you can choose the other alternative.

Payoffs

Your payoffs are determined by the SUM of your Round 1 payoff and *two times* your Round 2 payoff, for **1 of the 40 choice tasks** in the experiment. This task was selected at random, from the 40 tasks, before the experiment (we have the number corresponding to this task in a sealed envelope that we are happy to open for you when the experiment is over, if you wish). Observe that the Round 2 payoff counts double the Round 1 payoff.

This means that you can earn between 3 and 16 Euros, depending on your decisions in the 40 choice tasks, and your responses to the quiz. Now we will show you the conditions under which you could receive the minimum payoff (3 Euros) or alternatively the maximum payoff (16 Euros), as examples meant to illustrate how your payoff will be determined:

3 Euros:

Round 1 Payoff:	+ 1	Euro
Round 2 Payoff:	+ 1*2	Euros
Payoff on the Quiz:	+ 0	Euros
Total:	**3**	**Euros**

16 Euros:

Round 1 Payoff:	+ 5	Euros
Round 2 Payoff:	+ 5*2	Euros
Payoff on the Quiz:	+ 1	Euros
Total:	**16**	**Euros**

Once you finish the 40 choice tasks we ask that you remain in your seats until all of the other participants have also finished. At this point you should fill out the questionnaire so that we can calculate your payoffs. Once we have your payoffs ready we will call you up to the front of the room, individually, and we will give you your payoffs. We ask that you quietly gather your belongings before coming to the front of the room. Once you receive your payment you are free to leave. At this time you can also ask to see the contents of the sealed envelope containing the randomly selected number which determined which task you were paid for.

Summary

There are a total of 40 tasks. In each task there are two rounds. In each round you will choose one of two alternatives. The chosen alternative will generate a payoff for you in each round, but you will only observe this payoff for your Round 1 choice. The alternative that you choose in each round will give you an initial payoff of 1,2,3,4, or 5 Euros, but when we pay you at the conclusion of the experiment, the Round 2 payoff will be multiplied by 2. The probability distribution of obtaining each of these payoffs remains fixed for an alternative across the two rounds of a single choice task, but always differs from one alternative to the other. In addition, remember that from one choice task to the next, the probability distribution of each alternative always changes. Remember that in each choice task the relative position in which each alternative appears on your screen is selected at random, which means that nothing that you have observed in previous choice tasks is useful for choosing between the two alternatives in Round 1. Furthermore, remember that the payoff you observe after your choice in Round 1 teaches you something about the chosen alternative. Therefore, you can use this information when choosing an alternative in Round 2 of the same choice task. One of the 40 choice tasks is chosen at random in order to determine your payoffs. Your total payoff at the conclusion of the experiment will be your payoff in Round 1, plus twice your payoff in Round 2, plus up to 1 additional Euro if you respond correctly to the quiz. Thus, depending on your decisions, you can earn between 3 and 16 Euros. Once all of the participants have finished the experiment and the questionnaire, and we have calculated the payoffs for each participant, we ask that you wait silently for us to (individually) call you to the front of the room, at which point we will pay you. Once you receive your money you are free to leave.

Practice tasks

You will now participate in 3 non-paid practice tasks. These non-paid practice tasks *differ* from the paid tasks that follow in only three ways:

1) The choices that you make in these three tasks do not have any effect on what we pay you at the conclusion of the experiment.

2) After you have selected an alternative in Round 2 of each choice task we will show you a screen with generic probability distributions for each alternative from the task that you just finished. We show you this information in order to emphasize that the probabilities of each alternative do not change from Round 1 to Round 2 of the same choice task, but then they always change from one choice task to the next. You will only observe the payoff that you obtain in Round 1, just like in the paid tasks. Then, we will represent your Round 2 payoff with an X for the first task, with a Y for the second task, and with a Z for the third task. All you know is that X, Y, and Z are each equal to 1, 2, 3, 4, or 5, without knowing their exact payoffs. By contrast, in the 40 paid choice tasks we will not show you this screen with the generic probability distributions and payoffs. We show them to you here in order to emphasize what is the information that you know in each choice task, and what is the information that you do not know.

3) The probability distributions that we use in the practice tasks are NOT the same distributions that generate your payoffs in the 40 paid choice tasks. Once everybody has finished the practice tasks we will notify you when it is time to start the quiz. Then, following an opportunity to ask questions, you will start the 40 paid choice tasks.

You will now start the practice tasks.

You will now start the quiz.

TEST

1.- Suppose that in Round 1 of one of the choice tasks you choose one of the alternatives, for example, the one on the Left. Now, if in Round 2 of the same choice task you choose the same alternative, your payoff in Round 2:

a. may be different than your payoff in Round 1, but both payoffs were selected from the same probability distribution.

b. will always be the same as the Round 1 payoff.

c. will always be different than the Round 1 payoff.

Message: Correct/Incorrect. Your Round 2 payoff may be different than your payoff from Round 1, but both payoffs were generated by the same probability distribution. In particular, the Round 2 payoff does not necessarily have to be the same, or different, as the Round 1 payoff.

2.- Suppose that in Round 1 of one of the choice tasks, let's say Task 20, you have chosen one of the alternatives, let's say the one on the right, and you have received a certain payoff. Now, if in the next choice task (Task 21), you again choose the alternative on the right, either in Round 1 or in Round 2, then it follows that:

a. In Task 21 you will obtain the same payoff as in Task 20.

b. In Task 21 you will obtain the same payoff with the same probability as in Task 20.

c. None of the above, as the probability distribution over obtainable payoffs changes from one task to the next, and the payoff obtained in one choice task does not affect the probabilities of obtaining each possible payoff in a different choice task.

Message: Correct/ Incorrect. The probability distribution over obtainable payoffs changes from one task to the next, and the payoff obtained in one choice task does not affect the probabilities of obtaining each possible payoff in a different choice task.

3.- Which of the following statements is correct?

a. The Round 1 choice in each choice task is very important given that I can use the information from previous choice tasks in order to guess which alternative is more likely to generate a larger payoff.

b. Observing the payoff obtained in Round 1 of each task can be useful for the Round 2 choice in the same task, given that we know that if we select the same alternative in Round 2 as in Round 1 the distribution of payoffs is the same in Round 2 as it was in Round 1, while if we choose the other alternative, the distribution of payoffs for the alternative chosen in Round 2 will be different than it was in Round 1.

c. None of the above.

Message: Correct/Incorrect. The correct response is b, because in order for the distribution of payoffs in Round 2 to be the same as the distribution that generated the payoff in Round 1, then we must choose the same alternative, while in order for the distribution of payoffs in Round 2 to be different than the distribution that generated the payoff in Round 1, then we must choose the other alternative. Response a. is false because the distributions of both alternatives change from one choice task to the next, and the order of the alternatives is selected at random; therefore the information obtained from previous choice tasks is irrelevant for choosing in Round 1.

4. Which of the following statements is true?

a. Within a choice task, if I obtain a large payoff in Round 1, and then I choose the same alternative in Round 2, the probability of obtaining a large payoff in Round 2 is less than it was in Round 1. Therefore, if I choose the same alternative again in Round 2 it is very unlikely that I obtain another large payoff.

b. Within a choice task, if I obtain a small payoff in Round 1, and then I choose the same alternative in Round 2, the probability of obtaining a small payoff in Round 2 is less than it was in Round 1. Therefore, if I choose the same alternative again in Round 2 it is very unlikely that I obtain another small payoff.

c. None of the above.

Message: Correct/Incorrect. The correct response is c., none of the above. The statements in a. and b. are false because if you choose the same alternative in Rounds 1 and 2 of the same choice task then the distribution of payoffs in each round is exactly the same. In particular, if your Round 1 payoff was large this does not mean that the probability of getting a large payoff is then lower in Round 2 than it was in Round 1. Analogously, if your Round 1 payoff was small this does not mean that the probability of getting a small payoff is then lower in Round 2 than it was in Round 1.

Appendix F: Estimation and inference procedures

Parameter estimation by the EM algorithm

We denote the parameter estimates at the n^{th} iteration by

$$\Psi^{(n)} := \left(\pi_i^{(n)}, \left(\alpha_{i,k}^{(n)}\right)_{k=1}^{5}\right)_{i=1}^{N},$$

for $n = 0, 1, 2...$ Starting with some initial values $\Psi^{(0)}$, the algorithm iterates through the E- and M-step until the likelihood function $L(\Psi; \mathbf{Y}, \mathbf{X})$ converges.[61] In the E-step of the n^{th} iteration, we take the expected value of the complete-data log-likelihood l_c holding the current set of parameters fixed. Conditioned on previous iterates of the value parameter vector $\Psi^{(n-1)}$, and taking the data $(y_{j,t})_{t=1}^{40}$ and $(\boldsymbol{x}_{j,t})_{t=1}^{40}$ as fixed, $z_{i,j}$ is the only random variable which we integrate over to compute the expected value of the complete log-likelihood function, $\mathbb{E}_{\Psi^{(n-1)}}[l_c(\Psi; \mathbf{Y}, \mathbf{X})]$. Using Bayes' rule, and denoting the (conditional) expected value of $z_{i,j}$ given $\mathbf{Y}; \mathbf{X}$ and $\Psi^{(n-1)}$, we have

$$\begin{aligned}
\tau_{i,j}^{(n-1)} &= \mathbb{E}_{\Psi^{(n-1)}}[z_{i,j}|\mathbf{Y}, \mathbf{X}] \\
&= \mathbb{P}_{\Psi^{(n-1)}}[z_{i,j} = 1|\mathbf{Y}, \mathbf{X}] \\
&= \frac{\pi_i \left(\prod_{t=1}^{40} \left(p_{i,j,t}^{(n-1)}\right)^{y_{j,t}} \left(1 - p_{i,j,t}^{(n-1)}\right)^{1-y_{j,t}} \right)}{\sum_{i'=1}^{N} \pi_{i'} \left(\prod_{t=1}^{40} \left(p_{i',j,t}^{(n-1)}\right)^{y_{j,t}} \left(1 - p_{i',j,t}^{(n-1)}\right)^{1-y_{j,t}} \right)},
\end{aligned}$$

where $\mathbb{P}_{\Psi^{(n-1)}}[z_{i,j} = 1|\mathbf{Y}, \mathbf{X}]$ is the conditional probability that $z_{i,j} = 1$, given $\mathbf{Y}, \mathbf{X}$ and using the parameters $\Psi^{(n-1)}$; and

$$p_{i,j,t}^{(n-1)} = \frac{\exp\left(\boldsymbol{x}_{j,t} \cdot \boldsymbol{\alpha}_i^{(n-1)}\right)}{1 + \exp\left(\boldsymbol{x}_{j,t} \cdot \boldsymbol{\alpha}_i^{(n-1)}\right)}.$$

Thus, in the E-step on the n^{th} iteration, we compute

$$\begin{aligned}
Q\left(\Psi; \Psi^{(n-1)}\right) &:= \mathbb{E}_{\Psi^{(n-1)}}[l_c(\Psi; \mathbf{Y}, \mathbf{X})] \\
&= \sum_{j=1}^{100} \sum_{i=1}^{N} \tau_{i,j}^{(n-1)} \left(\log(\pi_i) + \sum_{t=1}^{40} \left(y_{j,t} \log(p_{i,j,t}) + (1 - y_{j,t}) \log(1 - p_{i,j,t})\right) \right).
\end{aligned}$$

In the M-step of the n^{th} iteration, we maximize $Q\left(\Psi; \Psi^{(n-1)}\right)$ with respect to the parameters Ψ, under the constraint $\sum_{i=1}^{N} \pi_i = 1$. The optimal values for $\boldsymbol{\pi}$, denoted by $\boldsymbol{\pi}^{(n)}$, are given by

$$\pi_i^{(n)} = \frac{\sum_{j=1}^{100} \tau_{i,j}^{(n-1)}}{100},$$

for $i = 1, ..., N$.

[61] When the likelihood function is multimodal, the EM algorithm estimates can be dependent on the choice of initial values. We choose to use 100 sets of random initial values through a process described in page 54 of McLachlan and Peel (2000). This process alleviates the effects of local maxima in the likelihood function. The convergence criterion is $\left| \left(L\left(\Psi^{(n)}; \mathbf{Y}, \mathbf{X}\right) - L\left(\Psi^{(n-1)}; \mathbf{Y}, \mathbf{X}\right)\right) / L\left(\Psi^{(n-1)}; \mathbf{Y}, \mathbf{X}\right) \right| < 10^{-2}$ (the default setting in the **flexmix R** package).

The roots of the derivatives of $Q\left(\Psi;\Psi^{(n-1)}\right)$ with respect to $\boldsymbol{\alpha}_i$, for $i=1,...,N$, cannot be solved analytically. Hence, $\left(\boldsymbol{\alpha}_i^{(n)}\right)_{i=1}^{N}$ are obtained using a Newton method or quasi-Newton method. We apply the EM algorithm through the **flexmix** R package (see, e.g., Grün and Leisch (2007) and Grün and Leisch (2008)) which utilizes the **optim** toolkit for general purpose optimization (see, e.g., R Core Team (2013)).

Confidence intervals and inference

It can be shown that under suitable regularity conditions, the maximum likelihood estimates of a mixture model are asymptotically normal (e.g., Redner and Walker (1984) and Example 4.2.2 of Amemiya (1985)). Thus applying either Theorem 3.1 of Redner and Walker (1984) or Theorem 4.2.4 of Amemiya (1985), we obtain that $\sqrt{J}\left(\hat{\Psi}-\Psi_0\right)$ converges to a normal distribution with mean $\mathbf{0}$ and covariance matrix $I\left(\Psi_0\right)^{-1}$ as J gets large, where Ψ_0 is a consistent root of the score function, and $I\left(\Psi_0\right)$ is the Fisher information matrix evaluated at Ψ_0.

Assuming that J is large enough for the asymptotics to apply, we can approximate the $(1-\alpha)\times 100\%$ confidence interval for the k^{th} element of Ψ_0 with

$$\left(\hat{\Psi}_k - z_{\alpha/2}\hat{I}\left(\hat{\Psi}\right)^{-1}_{kk}, \hat{\Psi}_k + z_{1-\alpha/2}\hat{I}\left(\hat{\Psi}\right)^{-1}_{kk}\right)$$

where $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are the $100\frac{\alpha}{2}$th and $100\left(1-\frac{\alpha}{2}\right)$th percentiles of the standard normal distribution, respectively, $\hat{\Psi}_k$ is the kth element of $\hat{\Psi}$, and $\hat{I}\left(\hat{\Psi}\right)^{-1}_{kk}$ is the kth row and kth column element of the matrix $\hat{I}\left(\hat{\Psi}\right)^{-1}$. Here $\hat{I}\left(\hat{\Psi}\right)$ is a consistent estimate of the Fisher information matrix which in our case is taken to be the numerical Hessian of the log-likelihood function, evaluated at $\hat{\Psi}$.

To compute the maximum likelihood estimate for $\theta_{i,k}$, we simply use the transformation

$$\hat{\theta}_{i,k} := \frac{e^{\hat{\alpha}_{i,k}}}{1+e^{\hat{\alpha}_{i,k}}}$$

for each i and k. Similarly, the confidence interval for $\theta_{i,k}$ are computed with the corresponding transformations of the lower and upper bounds of the interval for $\alpha_{i,k}$. These confidence intervals appear in Figure 2 and Table 2.

We can also compute the confidence intervals for a new observation in each of the classes. Let $(y_{J+1,t})_{t=1}^{R_k}$ and $(x_{J+1,t})_{t=1}^{R_k}$ be such a new observation where $z_{i,J+1}$ is known to equal 1, and $x_{J+1,t}=k$ for each t. The number of trials (i.e., the number of observations of the payoff k in the first round) is taken to be $R_k = \lfloor 40\mathbb{P}\left[x_{J+1,t}=k\right]\rfloor$; the floor of the expected number of times an individual experiences a payoff of k. Here, the floor operator is used to construct intervals which are conservative with respects to the number of trials.

The estimated proportion of switches for individual $J+1$ at payoff k is given as

$$\hat{q}_{i,J+1,k} := \frac{\sum_{t=1}^{R_k} y_{J+1,t}}{R_k}.$$

The estimated proportion has estimated mean $\hat{\theta}_{i,k}$ and variance $\hat{\theta}_{i,k}\left(1-\hat{\theta}_{i,k}\right)/R_k$ using a binomial argument. Thus, we can approximate the $100\,(1-\alpha)\,\%$ confidence interval of the true proportion of switching $q_{i,J+1,k}$ for the new individual with the approximate normal interval

$$\left(\max\left(0, \hat{q}_{i,J+1,k} - z_{\alpha/2}\sqrt{\frac{\hat{\theta}_{i,k}\left(1-\hat{\theta}_{i,k}\right)}{R_k}}\right), \min\left(1, \hat{q}_{i,J+1,k} + z_{1-\alpha/2}\sqrt{\frac{\hat{\theta}_{i,k}\left(1-\hat{\theta}_{i,k}\right)}{R_k}}\right)\right).$$

The probability that $x_{J+1,t} = k$ for each k is derived using Table 1. This is the procedure used to generate the intervals for Figure 3.

Hypothesis testing

In Section 5, we test two types of hypotheses, the *mean-type* H_0: $\mu_{i,\rho}^{(\bar{x})} - \mu_{i',\rho}^{(\bar{x})} \leq 0$ and the *variance-type* hypotheses H_0: $\mu_{i,\rho}^{(s^2)} - \mu_{i',\rho}^{(s^2)} \geq 0$, where $\rho \in \{sf, wf, ss, ws\}$ is the type of first or second-order stochastic dominance, and $i \neq i'$ are the classes between which we are testing. Under the mean-type null hypothesis, class i is said to have a smaller mean payoff than class i' within the tasks in T_ρ, and similarly under the variance-type null hypothesis, class i has greater payoff variance than class i' within the tasks in T_ρ.

In order to test these two types of hypotheses, we need to compute the sample statistics $\hat{\mu}_{i,\rho}^{(\bar{x})}$ and $\hat{\mu}_{i,\rho}^{(s^2)}$ for each i and ρ from the individual data $\bar{x}_{1,\rho}, ..., \bar{x}_{J,\rho}$ and $s_{1,\rho}^2, ..., s_{J,\rho}^2$, respectively. Because the parametric distributions of $\bar{x}_{1,\rho}, ..., \bar{x}_{J,\rho}$ and $s_{1,\rho}^2, ..., s_{J,\rho}^2$ are unknown, we use a two-sample permutation method to test the hypotheses non-parametrically.

The permutation tests applied are of the form described in Sections 3.6 and 3.7 of Good (2005) and implemented through the **perm R** package (see, e.g., Fay and Shaw (2010)). The steps for testing H_0: $\mu_{i,\rho}^{(\bar{x})} - \mu_{i',\rho}^{(\bar{x})} \leq 0$ are

1. Set the number of permutations M.
2. Compute the difference statistic $D = \hat{\mu}_{i,\rho}^{(\bar{x})} - \hat{\mu}_{i',\rho}^{(\bar{x})}$.
3. Combine the sample mean payoffs from classes i and i' into the set $S = \{\bar{x}_{j,\rho} : j \in J_i \cup J_{i'}\}$.
4. For $m = 1$ to M,

   (a) Randomly partition S into the sets S_i and $S_{i'}$ with $|J_i|$ and $|J_{i'}|$ observations from S, respectively.

(b) Compute the m^{th} permutation statistic $D_m = |J_i|^{-1} \sum_{j \in S_i} \bar{x}_{j,\rho} - |J_{i'}|^{-1} \sum_{j \in S_{i'}} \bar{x}_{j,\rho}$.

5. Estimate the p-value of the test by $(M+1)^{-1} \left(1 + \sum_{m=1}^{M} \mathbb{I}[D < D_m]\right)$ where

$$\mathbb{I}[D < D_m] = \begin{cases} 1 & \text{if } D > D_m, \\ 0 & \text{otherwise.} \end{cases}$$

For tests of the variance type, we simply replace step 2 and 4 (b) with $D = \hat{\mu}_{i,\rho}^{(s^2)} - \hat{\mu}_{i',\rho}^{(s^2)}$ and $D_m = |J_i|^{-1} \sum_{j \in S_i} s_{j,\rho}^2 - |J_{i'}|^{-1} \sum_{j \in S_{i'}} s_{j,\rho}^2$, respectively. Since the permutation test is Monte Carlo in nature, the obtained p-values are random, but the (unreported) 99% confidence intervals of the p-values reveal that all the tests computed in Section 5 remain significant when this randomness is taken into account.[62]

Appendix G: A test for learning across tasks

Pooled behavior

In order to test for possible learning effects across tasks, we perform a pooled analysis in which we partition the set of all the decisions into two halves. The first half contains tasks 1-20, and the second half tasks 21-40.

We then construct a contingency table of the number of decisions to "stay" with the same alternative, given the payoff experienced in the first round of a task, and the half that the decision was made in. The contingency table is presented in Table 8.

Table 8: Contingency table used to test for a change in behavior across the two halves of the experiment.

Half \ Payoff	1	2	3	4	5
FIRST	115	102	327	305	289
SECOND	91	96	309	306	300

Number of "stay" decisions, given the first round payoff and the half in which the decision was made.

Using the contingency table, we perform a χ^2-test for independence between the two factors (i.e., the rows and columns of the table). The test hypotheses are H_0: the decision to stay, based on a payoff, is *independent* of which half of the experiment the payoff occurs in; and H_1: the decision to stay, based on a payoff, is *dependent* on which half of the experiment the payoff occurs in.

The test statistic is 3.12 (4 degrees of freedom) and the p-value is 0.54. Thus, when pooling the entire group of 100 subjects, the half of the experiment that the decision is made in appears not to affect the decision.

[62] We constructed the confidence intervals using the **permTS** function from the **perm R** package. The number of permutations was set to $M = 999$ (the default value in **perm R**).

Individual tests

Now, we perform the same test of independence, but at an individual subject level. Here, however, instead of using the χ^2-test of the hypothesis, we use the Fisher exact-test method. This is because the χ^2-test relies on large sample assumptions, whereas the Fisher-exact test is small sample exact. We find that there are 0, 2, and 9 rejections (cumulative) of the null hypothesis at the 0.01, 0.05, and 0.1 confidence levels, respectively. A plot of the histogram of the p-values is given in Figure 7.

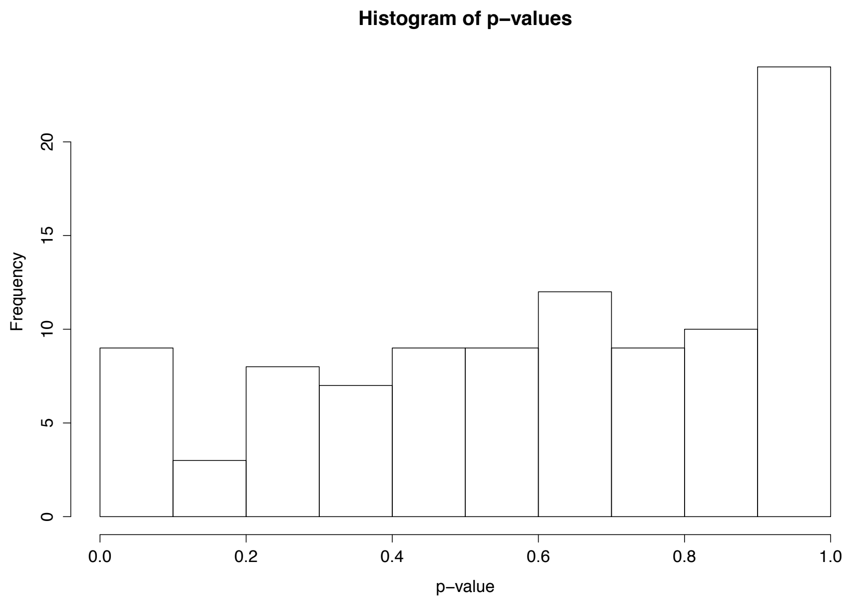

Figure 7: Histogram of all p-values from the 100 individuals.

Since we are testing 100 hypotheses simultaneously, the convention is to adjust the significance level to control the false discovery rate. Using a FDR (false discovery rate) control level of 0.05 and 0.1 (via the Benjamini and Hochberg (1995) method), we get no rejections. Thus, it is safe to conclude that there was no difference in behavior between the first and second half of the experiment.